A GENERAL INSPECTION-REPAIR-REPLACEMENT MODEL FOR SYSTEMS WITH UNOBSERVABLE STATES

by

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The undersigned certify that we have read a thesis, entitled 'A General Inspection-Repair-Replacement Model for systems with unobservable states' submitted to the Graduate School by Wan Siu Keung (溫兆強) in partial fulfillment of the requirement for the degree of Master of Philosophy in Statistics. We recommend that it be accepted.

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Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree of qualification of this or any other university or other institution of learning.

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Abstract

In this thesis, we consider a maintenance model for some system with unobservable states in which the states of the system can only be identified by inspections. After each inspection, if the system is identified as in the down state, a repair action will be taken. The system finally will be replaced by a new and identical one. An optimal maintenance model, called inspection-repair-replacement model, is studied in this thesis. An optimal policy is determined for minimizing the long-run expected cost per unit time . Under some mild conditions, an approximately optimal policy can be obtained numerically.

Contents

Chapter 1

Introduction and Review

1.1 Introduction

So far, most maintenance models assume that the state of a system is observable. However, in practice, it is not always the case. For example, weapons such as atomic bombs, missiles or shells are stored for an urgent need or an unexpected war. The top concern is whether they can be shot out at a critical moment. Of course, we are not willing to shoot out the atomic bombs for testing purpose. Instead, we inspect the system. Other examples happen in power supplies for a hospital or a steel manufacturing complex. In order to improve the reliability of a production process, a standby system is installed. If such a standby system fails to operate when required, it will cause a large amount of financial loss even a catastrophe. In normal case, we will not put the standby system into operation. The state of the standby system is also not observable. In these examples, we identify the system state by inspections.

The most research work in this topic has assumed a cost structure for inspection and repair, and derived inspection and repair policies which minimize the total expected cost of operation. Barlow and Proschan [1] described the average cost per unit time with perfect inspection and perfect repair. Wattanapanom and Shaw [17] studied a hazardous-inspection model for systems with exponential life distribution. Bulter [2] also dealt with a hazardous-inspection model in which the inspections have the potential of being harmful to the device. Thomas et al. [15] introduced a discrete Markov decision process model for periodic inspection and maintenance procedure that maximize the expected time until a catastrophe occurs. See Luss and Kander [5], Luss [6], Milioni and Pliska [7], [8], Nakagawa [9], Özekici and Papazyan [10], Özekici and Pliska [11], Rosenfield [12], Ross [14] for more references.

In application, the state of the standby system is usually not observable but can be identified only by inspection. Since the standby system is important to the prevention of a catastrophe, it is necessary to study an optimal maintenance policy such that the availability is kept in a high standard at all time and the long-run average cost per unit time is minimized. However, researches in inspection models so far have not paid enough attention to the area of availability or reliability.

To achieve this aim, Lam [3] has introduced a maintenance model for a standby system, in which the inspection-repair-replacement (IRR) policy is used. In his model, the failure of the system can be detected only through inspection. An optimal policy is a policy such that the availability of the system is always high enough while the long-run average cost per unit time is minimized. Lam [3] argued that the availability is more serious than the economical consideration. For a geometric model with an exponential distribution, he has suggested a simple algorithm for obtaining an optimal IRR policy.

In this thesis, we consider a more general IRR model in which some restrictions can be much relaxed. Moreover, we also try to merge inspection time, repair time if any and replacement time into our model. Under some mild conditions, an optimal IRR policy can be obtained numerically. In the next section, we will first review the IRR model introduced by Lam [3]. In Chapter 2, we will discuss the general IRR model. A numerical algorithm is introduced in Chapter 3. In Chapter 4, some numerical examples are considered. The sensitivity analysis is also discussed. Some conclusions and comments will be given in the last chapter.

1.2 Review

First of all, suppose that at the beginning a new system is installed. The system state at time t is a binary random variable,

$$X(t) = \begin{cases} 1 & \text{if the system is in the up state at time t,} \\ 0 & \text{if the system is in the down state at time t.} \end{cases}$$

Let the availability of the system at time t be

$$A(t) = \Pr(X(t) = 1) \quad \forall \ t \geq 0,$$

i.e., $A(t)$ is the probability that the system is up (in working condition) at time t.

Since the state of the system is not observable, one can identify the state through inspection only. After inspection, if the system is judged to be in the down state, we then repair the system so that it will recover from the down state, if the system is judged in the up state, we do nothing. Some time later the system will be replaced by a new and identical one.

In Lam's IRR model [3], an IRR policy $(t_1, t_2, \ldots, t_n, t_{r,n}; \alpha)$ is a maintenance policy in which $t_1, t_2, \ldots, t_n$ are the inspection times, while $t_{r,n}$ is the replacement time with $0 < t_1 < t_2 \ldots < t_n < t_{r,n}$, α is the lower bound of the availability so that

$$A(t) \geq \alpha \quad \forall \ t \geq 0. \tag{1.1}$$

Since an inspection may be imperfect, we can assume that an inspection identifies the down state correctly with probability p and incorrectly with probability $q = 1 - p$. Also, an inspection identifies the up state correctly with probability p' and incorrectly with probability $q' = 1 - p'$. Let $\theta_1, \theta_2, \ldots, \theta_n$ be the availabilities of the system at $t_1^*, t_2^*, \ldots, t_n^*$ respectively. Lam showed that $A(t_i^*) = \theta_i$, $p + q\theta_{i-1} \geq \alpha$, for $i = 1, 2, 3, \ldots, n$ and $A(t_{r,n}^*) = \alpha$, where $t_1^*, t_2^*, \ldots, t_n^*$ are the optimal inspection times and $t_{r,n}^*$ is the optimal replacement time.

Under the IRR policy $(t_1, t_2, \ldots, t_n, t_{r,n}; \alpha)$, if the system is identified in the down state, it will be repaired. Let I_i , C_i be the cost of the i^{th} inspection and the cost of i^{th} repair after the i^{th} inspection (if any), and let R_n be the replacement cost. Moreover, a penalty cost at rate $g(\alpha)$ is incurred at all time. It is reasonable to assume that the inspection cost I_i and the repair cost C_i are non-decreasing in i, and the replacement cost R_n is non-decreasing in n. Since the replacement cycles (the periods between two successive replacements) generate a renewal process, Lam suggested to choose an optimal IRR policy such that the constraint (1.1) is satisfied and the long-run average cost per unit time (or the average cost for simplicity) is minimized. By using the standard result in renewal reward process, the long run average cost per unit time is given by

$$\frac{\text{the expected cost incurred in a cycle}}{\text{the expected length of a cycle}}.$$

After some algebra, we have

$$C(t_1, t_2, \ldots, t_n, t_{r,n}; \alpha) = \left\{ \sum_{i=1}^{n} [I_i + (p - (p - q')\theta_i)C_i + R_n] \right\} / t_{r,n} + g(\alpha). \quad (1.2)$$

Since the determination of $(t_1, t_2, \ldots, t_n, t_{r,n}; \alpha)$ is equivalent to that of $(\theta_1, \theta_2, \ldots, \theta_n; \alpha)$, the average cost $C(t_1, t_2, \ldots, t_n, t_{r,n}; \alpha)$ can be denoted by $C_\alpha(\theta_1, \theta_2, \ldots, \theta_n)$. Generally speaking, (1.2) is a difficult problem because it is a very complicated nonlinear programming problem. However, Lam [3] studied a geometric model with the exponential distribution, an optimal solution can be obtained through some simple algorithm easily.

In this model, let X_i be the lifetime of the system after the $(i-1)^{th}$ inspection and repair (if any). Assume that $X_i \mid X_i > 0, \;\; i = 1, 2, 3, \ldots$ follow a distribution F_i, where $F_i(x) = F(a^{i-1}x)$ for some $a \geq 1$. Suppose that F is the exponential distribution $\text{Exp}(\lambda)$ with density

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 1. (Lam [3])

For the geometric model with the exponential distribution, if

$$a(R_{n+2} - R_{n+1}) \geq R_{n+1} - R_n, \quad \text{for } n = 1, 2, \ldots$$

then the minimum long-run average cost per unit time is given by

$$\min C_\alpha(\theta_1, \theta_2, \ldots, \theta_n) = \min_{\theta \geq \alpha} C_\alpha(\underbrace{\theta, \theta, \ldots, \theta}_{M(\theta) \ terms}) = \min_{\theta \geq \alpha} C_\alpha(M(\theta)) \tag{1.3}$$

where

$$M(\theta) = \min\{m : d_m \geq 0\} \tag{1.4}$$

with

$$\begin{aligned} d_m &= a^{m+1}\left[I_{m+1} + \delta C_{m+1} + R_{m+1} - R_m\right] ln \left[\frac{p + q\theta}{\theta^a}\right] - \left\{(a-1)\sum_{i=1}^{m} I_i + aI_{m+1} \right. \\ &\left. + \delta\left[(a-1)\sum_{i=1}^{m} C_i + aC_{m+1}\right] + aR_{m+1} - R_m\right\} ln \left[\frac{p+q\theta}{\theta}\right] \\ \delta &= p - (p - q')\theta \ \blacksquare \end{aligned}$$

Theorem 1 is utilized to derive a finite algorithm for an optimal IRR policy. Furthermore, if the value of θ is allowed to be in an interval [L, U] $\subset (0, 1)$, a numerical algorithm was suggested by Lam. The algorithm can be described as follows.

1. Divide interval [L, U] into k equally spaced subintervals

$$L = \theta_{(0)} \leq \theta_{(1)} \leq \theta_{(2)} \cdots \leq \theta_{(k)} = U.$$

2. For each $\theta = \theta_{(i)}$, $i = 0, 1, 2, \ldots, k$, determine $M_i = M(\theta_{(i)})$ from (1.4). Then evaluate the values $C_\alpha(M(\theta_{(i)}))$.

3. By comparing the values of $C_\alpha(M(\theta_{(i)}))$, $i = 1, 2, \ldots, k$, choose the smallest value as the minimum $C_\alpha(M^*)$, the corresponding policy $(t_1^*, t_2^*, \ldots, t_n^*, t_{r,n}^*; \alpha)$ is an approximately optimal IRR policy.

Lam's IRR model is an interesting model and should have a potential application especially in the maintenance problem of a system with unobservable states or the maintenance problem for a standby system. In Lam's model, under the condition that the availability of the system is high enough at all times, an optimal IRR policy is determined such that the long-run average cost per unit time (or the average cost for simplicity) is minimized.

Nevertheless, there are some shortcomings in Lam's IRR model. Firstly, the model assumes that the conditional lifetime distributions F_i satisfy $F_i(x) = F(a^{i-1}x)$ for some $a \geq 1$ and F is the exponential distribution $\text{Exp}(\lambda)$. This assumption seems to be too strong. Secondly, the inspection time, the repair time, and the replacement time are all negligible. It looks not realistic in real situation. Consequently, the scope of application is then limited due to these two strong assumptions. To overcome these weak points, we shall study a general IRR model in the next chapter.

Chapter 2

A General IRR model for a system with unobservable states

2.1 Model and Assumptions

We study a general IRR model by making the following assumptions.

Assumption 1.

Assume that at the beginning, a new system is installed. It will be replaced by an identical new one sometime later. Let X_1 be the time period of the system being in the up state since the installation or the last replacement. In general, for $i > 1$, let X_i be the time period that the system being in the up state after the $(i-1)^{th}$ inspection, and repair if any. Assume further that

(1) $\Pr(X_1 = 0) = 0$,

(2) $X_i \mid X_i > 0 \sim F_i \quad i = 1, 2, 3 \ldots,$

(3) $\{F_i \ , \ i = 1,2,\ldots\}$ is stochastically decreasing in the sense that for all

$x > 0$ and $i = 1,2,\ldots$, $F_i(x) \leq F_{i+1}(x)$. ∎

Assumption 2.

An IRR policy $(s_1, s_2, \ldots, s_n, s_{r,n}; \theta)$ is applied, in which s_i, $i = 1,2,\ldots,n$, is the time interval between the completion of the $(i-1)^{th}$ inspection(and repair if any) and the i^{th} inspection, $s_{r,n}$ is the time interval between the completion of the n^{th} inspection(and repair if any) and the replacement followed. Let t_i be the time of i^{th} inspection, and $t_{r,n}$ be the replacement time. Then $0 < t_1 < t_2 \ldots < t_n < t_{r,n}$. Let θ be the availability of the system at times $t_1, t_2, \ldots, t_n$ and $t_{r,n}$, which is assumed not less that α, i.e.,

$$\begin{cases} A(t_i) & = \theta \quad i = 1,2,\ldots,n, \\ A(t_{r,n}) & = \theta, \\ \theta & \geq \alpha. \end{cases} \tag{2.1}$$

An inspection can identify the down state correctly with probability p and incorrectly with probability $q = 1 - p$. Also, an inspection can identify the up state correctly with probability p' and incorrectly with probability $q' = 1 - p'$. Assume that an inspection does not change the system state and a repair is always effective. ∎

Assumption 3.

Let the i^{th} inspection time be Y_i, and let the repair time followed (if any) be Z_i and the replacement time be W. Denote $E(Y_i) = \lambda_i$, $E(Z_i) = \mu_i$, and $E(W) = \tau$. Assume further that λ_i, μ_i are both non-decreasing in i. ∎

Assumption 4.

Under an IRR policy $(s_1, s_2, \ldots, s_n, s_{r,n}; \theta)$, we assume that the cost of the i^{th} inspection is U_i , the cost of the repair after the i^{th} inspection (if any) is V_i, while the replacement cost is R. Moreover, a penalty cost with rate $g(\theta)$ is incurred at all times. Denote $E(U_i) = I_i$, $E(V_i) = C_i$. Assume further that I_i and C_i are both non-decreasing in i. $g(\theta)$ is non-increasing in θ. ■

Assumption 5.

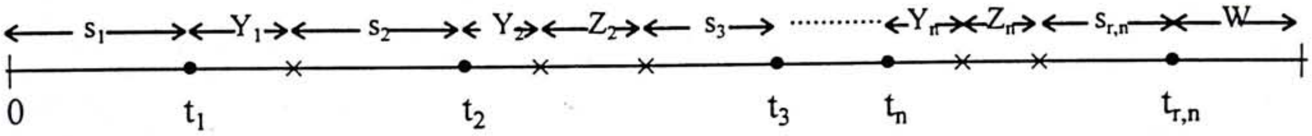
Let $\omega = p - (p - q')\theta$, then the following condition is satisfied:

$$\frac{\lambda_i + \omega\mu_i}{I_i + \omega C_i} \geq \frac{\lambda_{i+1} + \omega\mu_{i+1}}{I_{i+1} + \omega C_{i+1}} \quad \text{for all } i. \quad \blacksquare$$

Now, we make some remarks to the assumptions of our model. Assumption 1 depicts the structure of lifetime distribution of the system as well. Part (1) is trivial since a new system is installed at the beginning. In Part (2), we consider the conditional distribution of X_i, given that $X_i > 0$, instead of the distribution X_i itself because we assume that a repair is always effective such that once a repair is carried out the system will be in the up state for a certain non-zero time period. Thus, the event $X_i > 0$ is equivalent to that the system is in up state after $(i-1)^{th}$ inspection and repair, if any. Part (3) of Assumption 1 is also plausible because of the system deterioration and the aging effect. An inspection plus a repair (if any) is an interruption of the system. An interruption may destroy the regular functioning condition of the system and may cause the system deterioration.

In Lam's IRR model, the availability at inspection times are not fixed but by no means less than α. However, Theorem 1 shows that, by using an optimal policy, the availability must be the same at all inspection times. Although this result is only applied to the case of the geometric model with the exponential distribution, it enlightens us on the study of the general IRR model, we can consider a simpler case that all the availability at inspection times and replacement times are all equal to a constant θ and $\theta \geq \alpha$. Eventually, this restriction of equal availability at inspection times can ease the minimization problem a lot. Therefore, the assumption of equal availability at all inspection times, is made in Assumption 2.

In Lam's IRR model, the inspection times, the repair times if any, and the replacement time were assumed to be negligible. As mentioned in Chapter 1, it doesn't match the real situation and should be overcome by introducing a more general assumption. In Assumption 3, we assume that the i^{th} inspection takes Y_i unit of time, the i^{th} repair (if any) takes Z_i unit of time, and the replacement lasts W unit of time.



The above graph shows a realization of the system process. At the beginning, a new system is installed. The system is inspected at time t_1 $(= s_1)$ and the inspection takes Y_1 units of time. The inspection identifies it as up, therefore no repair is made. After s_2 time units, the second inspection takes place at time t_2. Now, the inspection lasts for Y_2 time units and identifies it as in down state. A repair follows and it lasts Z_2 units of time. This procedure continues until the completion of the n^{th} inspection which takes Y_n time units, and followed by a repair for Z_n time units. Afterwards, the system is replaced by an identical new one $s_{r,n}$ time units later, and it takes W units of time. Note that the general IRR policy defined here is different from that defined in Lam's paper [3]. Here, the IRR policy is defined on the basis of time interval between completion of inspection (and repair if any) and the next inspection. However, in Lam's IRR model, since the inspection and repair time are all negligible, we can directly define the IRR policy based on the inspection times.

It is easy to see that t_i, s_i, Y_i, Z_i, W satisfy the following equations:

$$\begin{cases} t_i - t_{i-1} &= Y_{i-1} + Z_{i-1} \cdot \psi_{i-1} + s_i \quad i = 1, 2, \ldots n, \\ t_{r,n} - t_n &= Y_n + Z_n \cdot \psi_n + s_{r,n}, \end{cases} \tag{2.2}$$

where

$$\psi_i = \begin{cases} 1 & \text{if a repair takes place after the } i^{th} \text{ inspection,} \\ 0 & \text{otherwise,} \end{cases}$$

with $t_0 = 0$, $Y_0 = 0$ and $Z_0 = 0$.

In Assumption 4, $E(U_i)$, $E(V_i)$ are non-decreasing function in i. According to our practical experience, it seems true that the later the inspection(and/or repair) taken, the more the complexity accumulated, and the higher the cost incurred. The penalty cost rate $g(\theta)$ would be non-increasing in θ, due to the fact that the higher the availability θ adopted, the less the penalty charged.

Note that the inspection cost and repair cost defined in the general IRR model is also different from that defined in Lam's paper [3]. In Lam's IRR model, inspection cost I_i and repair cost C_i were assumed to be known and non-decreasing in i. Here, we adopt a more general case that the inspection cost and repair cost are stochastic in nature.

Further, inspection cost and repair cost may be correlated to the time length of inspection and repair respectively. Therefore, as a special case, we may assume that the inspection cost and repair cost are proportional to the time length of inspection and repair. Under this setting, we may have the following assumptions.

Assumption 4$'$.

Under an IRR policy $(s_1, s_2, \ldots, s_n, s_{r,n}; \theta)$, we assume that the inspection cost and repair cost are proportional to the inspection time and the repair time. Let the inspection cost rate be I, and the repair cost rate be C. Then the cost of the i^{th} inspection is $U_i = IY_i$ and the cost of the repair after i^{th} inspection (if any) is $V_i = CZ_i$. The replacement cost is R. Furthermore, a penalty cost with rate $g(\theta)$ is incurred at all times. ▪

Assumption 5$'$.

Assume that one of the following conditions satisfies:

(1) $C \geq I$ and $\frac{\lambda_i}{\mu_i} \geq \frac{\lambda_{i+1}}{\mu_{i+1}}$ for all i,

(2) $C \leq I$ and $\frac{\lambda_i}{\mu_i} \leq \frac{\lambda_{i+1}}{\mu_{i+1}}$ for all i. ▪

It is easy to show that Assumptions 4' and 5' imply Assumptions 4 and 5. Readers may wonder the rationale behind Assumption 5 but it will come clear later. However, the interpretation of the conditions in Assumption 5$'$ are quite obvious. Condition (1) means that the repair cost rate is higher than the inspection cost rate, and the expected repair time increases faster than the expected inspection time. A similar interpretation can be applied to Condition (2).

2.2 Long-run average cost per unit time incurred in a replacement cycle

Clearly, the replacement cycles, i.e., the periods between the completion of two successive replacements will generate a renewal process. Thus under the IRR policy $(s_1, s_2, \ldots, s_{r,n}; \theta)$, by applying the standard result in renewal reward process(See Ross [13]), the long-run average cost per unit time $C(s_1, s_2, \ldots, s_n, s_{r,n}; \theta)$ is given by

$$\frac{\text{the expected cost incurred in a cycle}}{\text{the expected length of a cycle}}. \tag{2.3}$$

Our aim is to determine an optimal IRR policy such that the long-run average cost per unit time is minimized. To this end, at first we note that $s_{r,n} = s_{n+1}$. Followed from (2.2), the denominator of (2.3) is given by

$$\begin{aligned} T_{r,n} &= E(t_{r,n} + W) \\ &= E[\sum_{i=1}^{n}(Y_{i-1} + Z_{i-1} \cdot \psi_{i-1} + s_i) \quad + Y_n + Z_n \cdot \psi_n + s_{r,n} + W)] \\ &= \sum_{i=1}^{n} s_i + \sum_{i=1}^{n} [E(Y_i) + E(Z_i) \cdot E(\psi_i)] + s_{r,n} + \tau \\ &= \sum_{i=1}^{n+1} s_i + \sum_{i=1}^{n} (\lambda_i + \omega\mu_i) + \tau \end{aligned} \tag{2.4}$$

where

$\omega = E(\psi_i)$

$=$ Pr(i^{th} inspection identifies the system as in down state)

$$= \Pr(i^{th} \text{ inspection identifies the system as in down state} \mid X(t_i) = 1) \cdot \Pr(X(t_i) = 1)$$

$$+ \Pr(i^{th} \text{ inspection identifies the system as in down state} \mid X(t_i) = 0) \cdot \Pr(X(t_i) = 0)$$

$$= \theta q' + (1 - \theta)p = p - (p - q')\theta.$$

The numerator of (2.3) includes the expected inspection cost, the expected repair cost, the expected penalty cost incurred in a cycle and the replacement cost. It is equal to

$$\begin{aligned} & E\left\{\sum_{i=1}^{n}[U_i + V_i \cdot \psi_i] + R + g(\theta)(t_{r,n} + W)\right\} \\ = & \sum_{i=1}^{n}[E(U_i) + E(V_i) \cdot E(\psi_i)] + R + g(\theta) \cdot T_{r,n} \\ = & \sum_{i=1}^{n}[I_i + \omega C_i] + R + g(\theta) \cdot T_{r,n}. \end{aligned}$$

Thus, the long-run average cost per unit time is given by

$$C(s_1, s_2, \ldots, s_n, s_{r,n}; \theta) = \left\{\sum_{i=1}^{n}[I_i + \omega C_i] + R\right\} / T_{r,n} + g(\theta). \tag{2.5}$$

In Chapter 3, a finite algorithm for minimizing the objective function (2.5) will be discussed.

Chapter 3

The Algorithm

3.1 The Key Point and the Local Turning Point of the average cost

Our problem so far is to minimize $C(s_1, s_2, \ldots, s_n, s_{r,n}; \theta)$ among all IRR policies $(s_1, s_2, \ldots, s_n, s_{r,n}; \theta)$. It is a complicated nonlinear programming problem and is not easy to solve analytically. An alternative way is to find an algorithm so that the minimization problem can be solved numerically.

According to Assumption 2, we have $A(t_1) = A(s_1)$, and

$$\begin{aligned} \theta &= \Pr(X(s_1) = 1) \\ &= \Pr(X_1 > s_1 \mid X(t_0) = 1) \cdot \Pr(X(t_0) = 1) \\ &= \bar{F_1}(s_1). \end{aligned}$$

Thus

$$s_1 = F_1^{-1}(1 - \theta), \tag{3.1}$$

where $\bar{F}_1$ is the survival function of X_1, and F_1^{-1} is the inverse function of F_1.

In general, from (2.1) and Assumption 1, we have,

$$\begin{aligned}
\theta &= \Pr(X(t_i) = 1) \\
&= \Pr(X(t_i) = 1 \mid X(t_{i-1}) = 1) \cdot \Pr(X(t_{i-1}) = 1) \\
&\quad + \Pr(X(t_i) = 1 \mid X(t_{i-1}) = 0) \cdot \Pr(X(t_{i-1}) = 0) \\
&= \Pr(X_i > s_i \mid X(t_{i-1} + Y_{i-1} + Z_{i-1} \cdot \psi_{i-1}) = 1) \cdot \theta \\
&\quad + \Pr(X_i > s_i \mid X(t_{i-1} + Y_{i-1} + Z_{i-1}) = 1) \cdot p(1-\theta) \\
&= \bar{F}_i(s_i) \cdot (p + q\theta). \qquad (3.2)
\end{aligned}$$

Thus

$$s_i = F_i^{-1}(1 - \frac{\theta}{p + q\theta}). \qquad (3.3)$$

Similarly, due to the fact $s_{r,n} = s_{n+1}$, we have

$$s_{r,n} = F_{n+1}^{-1}(1 - \frac{\theta}{p + q\theta}). \qquad (3.4)$$

On the basis of (3.1.), (3.3) and (3.4.), $s_1, s_2, \ldots, s_n$ and $s_{n,r}$ can be determined if $F_i, \quad i = 1, 2, \ldots$ and θ are known. Therefore, we can use $C(n, \theta)$ to denote the average cost, i.e.,

$$C(n, \theta) \;=\; C(s_1, s_2, \ldots, s_n, s_{r,n}; \theta).$$

Theorem 2.

Under the IRR policy $(s_1, s_2, \ldots, s_n, s_{r,n}; \theta)$, the availability $A(t) \geq \alpha$ for $0 \leq t \leq t_{r,n}$. ∎

Proof.

We first consider the availability at time t which satisfies

$$t_{i-1} + Y_{i-1} + Z_{i-1} \cdot \psi_{i-1} \leq t \leq t_i, \quad i = 1, 2, \ldots, n+1,$$

with $t_{n+1} = t_{r,n}$. Let

$$d_i = t - (t_{i-1} + Y_{i-1} + Z_{i-1} \cdot \psi_{i-1}) \ , \quad i = 1, 2, \ldots, n+1,$$

then $d_i \leq s_i$. From (3.2), we have

$$\begin{aligned} A(t) &= \bar{F}_i(d_i) \cdot (p + q\theta) \\ &\geq \bar{F}_i(s_i) \cdot (p + q\theta) \\ &= \theta \geq \alpha \end{aligned}$$

If t is the time in an inspection period or a repair period, according to Assumption 2, an inspection does not change the system state and a repair is always effective, the availability is the same as at the beginning of the inspection. Therefore, $A(t) \geq \alpha$ for all $0 \leq t \leq t_{r,n}$. ∎

Lemma 1.

$\{s_n, n = 1, 2, \ldots\}$ is an non-increasing sequence. ∎

Proof.

Since $\{F_i, \ i = 1, 2, \ldots\}$ is stochastically decreasing, $F_i^{-1}(x) \geq F_{i+1}^{-1}(x)$. Then (3.1) implies that

$$s_1 = F^{-1}(1 - \theta) \geq F^{-1}(1 - \frac{\theta}{p + q\theta}) \geq F_2^{-1}(1 - \frac{\theta}{p + q\theta}) = s_2.$$

In general,

$$s_i = F_i^{-1}(1 - \frac{\theta}{p + q\theta}) \geq F_{i+1}^{-1}(1 - \frac{\theta}{p + q\theta}) = s_{i+1} \quad \forall \text{ i.}$$ ∎

By noting that $s_{r,n} = s_{n+1}$, we have

$$s_1 \geq s_2 \geq s_3 \cdots \geq s_n \geq s_{r,n} = s_{n+1}. \tag{3.5}$$

Now (2.5) can be rewritten as

$$\begin{aligned} C(n,\theta) &= \frac{\sum_{i=1}^{n}[I_i + \omega C_i] + R}{T_{r,n}} + g(\theta) \\ &= \frac{B_n}{T_{r,n}} + g(\theta), \end{aligned}$$

where $B_n = \sum_{i=1}^{n}[I_i + \omega C_i] + R$. Denote $\Delta(n,\theta) = C(n+1,\theta) - C(n,\theta)$, then

$$\Delta(n,\theta) = \frac{\delta_n}{T_{r,n} \cdot T_{r,n+1}},$$

where

$$\begin{aligned} \delta_n &= B_{n+1} \cdot T_{r,n} - B_n \cdot T_{r,n+1} \\ &= \left[\sum_{i=1}^{n+1}(I_i + \omega C_i) + R\right] \cdot \left[\sum_{i=1}^{n+1} s_i + \sum_{i=1}^{n}(\lambda_i + \omega\mu_i) + \tau\right] \\ &\quad - \left[\sum_{i=1}^{n}(I_i + \omega C_i) + R\right] \cdot \left[\sum_{i=1}^{n+2} s_i + \sum_{i=1}^{n+1}(\lambda_i + \omega\mu_i) + \tau\right] \\ &= (I_{n+1} + \omega C_{n+1}) \cdot \left(\sum_{i=1}^{n+1} s_i + \sum_{i=1}^{n}(\lambda_i + \omega\mu_i) + \tau\right) \\ &\quad - \left(\sum_{i=1}^{n}(I_i + \omega C_i)\right) \cdot (s_{n+2} + \lambda_{n+1} + \omega\mu_{n+1}) \\ &\quad -R\left(s_{n+2} + \lambda_{n+1} + \omega\mu_{n+1}\right). \end{aligned} \tag{3.6}$$

Then

$$\begin{aligned} &\delta_{n+1} - \delta_n = (I_{n+2} + \omega C_{n+2}) \cdot \left(\textstyle\sum_{i=1}^{n+2} s_i + \sum_{i=1}^{n+1}(\lambda_i + \omega\mu_i) + \tau\right) \\ &- (I_{n+1} + \omega C_{n+1}) \cdot \left(\textstyle\sum_{i=1}^{n+1} s_i + \sum_{i=1}^{n}(\lambda_i + \omega\mu_i) + \tau\right) \\ &- \left(\textstyle\sum_{i=1}^{n+1}(I_i + \omega C_i)\right) \cdot (s_{n+3} + \lambda_{n+2} + \omega\mu_{n+2}) \\ &+ \left(\textstyle\sum_{i=1}^{n}(I_i + \omega C_i)\right) \cdot (s_{n+2} + \lambda_{n+1} + \omega\mu_{n+1}) \\ &+ R(s_{n+2} - s_{n+3}) + R[(\lambda_{n+1} - \lambda_{n+2}) + \omega(\mu_{n+1} - \mu_{n+2})]. \end{aligned} \tag{3.7}$$

After some algebra, we can show that

$$\delta_{n+1} - \delta_n = A_1 + A_2 + A_3 + A_4,$$

where

$$\begin{aligned}
A_1 &= (I_{n+2} + \omega C_{n+2} + R)s_{n+2} - (I_{n+1} + \omega C_{n+1} + R)s_{n+3}, \\
A_2 &= \sum_{i=1}^{n}(I_i + \omega C_i)(s_{n+2} - s_{n+3}), \\
A_3 &= \left[\sum_{i=1}^{n}(\lambda_i + \omega\mu_i)\right](I_{n+2} + \omega C_{n+2} - I_{n+1} - \omega C_{n+1}) \\
&\quad - \left[\sum_{i=1}^{n}(I_i + \omega C_i)\right](\lambda_{n+2} + \omega\mu_{n+2} - \lambda_{n+1} - \omega\mu_{n+1}) \\
&\quad + (I_{n+2} + \omega C_{n+2})(\lambda_{n+1} + \omega\mu_{n+1}) - (I_{n+1} + \omega C_{n+1})(\lambda_{n+2} + \omega\mu_{n+2}), \\
A_4 &= \left(\sum_{i=1}^{n+1} s_i + \tau\right)[(I_{n+2} - I_{n+1}) + \omega(C_{n+2} - C_{n+1})] \\
&\quad - R\left[(\lambda_{n+2} - \lambda_{n+1}) + \omega(\mu_{n+2} - \mu_{n+1})\right].
\end{aligned}$$

First of all, it is easy to see that A_1 and $A_2 \geq 0$ since $I_{n+2} \geq I_{n+1}$, $C_{n+2} \geq C_{n+1}$ and $s_{n+2} \geq s_{n+3}$. In the next place, we are going to find some conditions so that A_3 and A_4 are greater than zero, and $\delta_{n+1} - \delta_n \geq 0$ consequently.

To this end, let $h_i = \lambda_i + \omega\mu_i$, $k_i = I_i + \omega C_i$, then h_i and k_i are both non-decreasing in i. Moreover, Assumption 5 implies that $\dfrac{h_i}{k_i} \geq \dfrac{h_{i+1}}{k_{i+1}} \ \forall \ i$.

Consider

$$\begin{aligned}
\frac{h_{n+1}}{k_{n+1}} - \frac{h_{n+2} - h_{n+1}}{k_{n+2} - k_{n+1}} &= \frac{h_{n+1}k_{n+2} - h_{n+2}k_{n+1}}{k_{n+1}(k_{n+2} - k_{n+1})} \\
&= \frac{k_{n+2}}{k_{n+2} - k_{n+1}}\left(\frac{h_{n+1}}{k_{n+1}} - \frac{h_{n+2}}{k_{n+2}}\right) \geq 0.
\end{aligned}$$

Therefore, under Assumption 5,

$$\frac{h_i}{k_i} \geq \ldots \geq \frac{h_{n+1}}{k_{n+1}} \geq \frac{h_{n+2} - h_{n+1}}{k_{n+2} - k_{n+1}} \quad \text{for } i = 1, 2, \ldots, n. \tag{3.8}$$

Lemma 2.

Under Assumption 5, $A_3 \geq 0$. ∎

Proof.

$$
\begin{aligned}
A_3 &= \left[\sum_{i=1}^{n}(\lambda_i + \omega\mu_i)\right][(I_{n+2} + \omega C_{n+2}) - (I_{n+1} + \omega C_{n+1})] \\
&\quad - \left[\sum_{i=1}^{n}(I_i + \omega C_i)\right][(\lambda_{n+2} + \omega\mu_{n+2}) - (\lambda_{n+1} + \omega\mu_{n+1})] \\
&\quad + (I_{n+2} + \omega C_{n+2})(\lambda_{n+1} + \omega\mu_{n+1}) - (I_{n+1} + \omega C_{n+1})(\lambda_{n+2} + \omega\mu_{n+2}) \\
&= \left[\sum_{i=1}^{n} h_i\right](k_{n+2} - k_{n+1}) - \left[\sum_{i=1}^{n} k_i\right](h_{n+2} - h_{n+1}) + k_{n+2}h_{n+1} - k_{n+1}h_{n+2} \\
&= \sum_{i=1}^{n}\left\{k_i(k_{n+2} - k_{n+1})\left[\frac{h_i}{k_i} - \frac{h_{n+2} - h_{n+1}}{k_{n+2} - k_{n+1}}\right]\right\} + k_{n+1}k_{n+2}\left[\frac{h_{n+1}}{k_{n+1}} - \frac{h_{n+2}}{k_{n+2}}\right] \\
&\geq 0.
\end{aligned}
$$

The last inequality is due to (3.8). ∎

Lemma 3.

Under Assumption 5, let

$$
n(\theta) = \min\left\{n > 0 : \sum_{i=1}^{n+1} s_i + \tau \geq R\left(\frac{\lambda_n + \omega\mu_n}{I_n + \omega C_n}\right)\right\}, \tag{3.9}
$$

then for any $n \geq n(\theta)$, $A_4 \geq 0$. ∎

Proof.

For any $n \geq n(\theta)$, it follows from (3.9) and Assumption 5 that

$$
\sum_{i=1}^{n+1} s_i + \tau \geq \sum_{i=1}^{n(\theta)+1} s_i + \tau \geq R\left(\frac{\lambda_{n(\theta)} + \omega\mu_{n(\theta)}}{I_{n(\theta)} + \omega C_{n(\theta)}}\right) \geq R\left(\frac{\lambda_n + \omega\mu_n}{I_n + \omega C_n}\right) = \frac{Rh_n}{k_n}. \tag{3.10}
$$

Then (3.10) yields that for $n \geq n(\theta)$,

$$
\begin{aligned}
A_4 &\geq R\left[\frac{h_n}{k_n}(k_{n+2} - k_{n+1}) - (h_{n+2} - h_{n+1})\right] \\
&= R(k_{n+2} - k_{n+1})\left[\frac{h_n}{k_n} - \frac{h_{n+2} - h_{n+1}}{k_{n+2} - k_{n+1}}\right] \geq 0.
\end{aligned}
$$

Once again, the last inequality is due to (3.8). ∎

Note that in (3.9), the left hand side of the inequality is nondecreasing, while the right hand side is nonincreasing, therefore there exists an unique integer $n(\theta)$ (may be ∞) such that (3.9) holds.

By Lemmas 2 and 3, it is easy to verify that $\delta_{n+1} - \delta_n \geq 0$ for all $n \geq n(\theta)$. Consequently, δ_n is increasing for all $n \geq n(\theta)$. Now, we can define

$$m(\theta) = \min\{n : n \geq n(\theta), \quad \delta_n \geq 0\}. \tag{3.11}$$

Note that $m(\theta)$ is the smallest integer n such that $\delta_n \geq 0$. For any $n(\theta) \leq n < m(\theta)$, $\delta_n < 0$ implies $C(n+1,\theta) < C(n,\theta)$ and an optimal solution has not been attained. For any $n \geq m(\theta)$, since $\delta_n \geq 0$ and is increasing which further implies $C(n+1,\theta) \geq C(n,\theta)$. Thus, $C(m(\theta),\theta)$ is the local minimum of the average cost for $n \geq n(\theta)$ and fixed availability θ. Therefore, $m(\theta)$ may be called the local turning point of the average cost. From (3.11), it is clear that if $\delta_{n(\theta)} \geq 0$, then $m(\theta) = n(\theta)$.

Therefore, in searching for the global minimum of the average cost for a given availability θ, we can compare the average cost $C(m(\theta),\theta)$ with the $\min\limits_{n \leq n(\theta)} C(n,\theta)$. Obviously, $n(\theta)$ is the key point for determining the minimum of the average cost $C(n,\theta)$. In fact, we have shown the following theorem.

Theorem 3.

Under Assumptions 1-5, we have

$$\min_{n,\theta} C(n,\theta) = \min_{\theta}\left[\min\left(\min_{n \leq n(\theta)} C(n,\theta), C(m(\theta),\theta)\right)\right] \tag{3.12}$$

where $n(\theta)$ and $m(\theta)$ are defined by (3.9) and (3.11) respectively. ∎

3.2 A Finite Algorithm

By using Theorem 3, a finite algorithm for minimizing $C(n,\theta)$ can be adopted in the following way.

(1) Divide interval $[\alpha, 1]$ into k equally spaced subintervals

$$\alpha = \theta_0 \leq \theta_1 \leq \theta_2 \cdots \leq \theta_k < 1$$

θ_k is less than 1, instead of being equal to 1. Otherwise, if $\theta_k = 1$, then from (3.1), (3.3) and (3.4), we will see that $s_i, \quad i = 1, 2, \ldots, n$ and $s_{\tau,n}$ are all 0. The IRR policy is $(0, 0, \ldots, 0; 1)$, it is unrealistic and must be excluded.

(2) Start with $i = 0$, for given value θ_i, determine $s_1, s_2, \ldots, s_{\tau,n}$ from (3.1), (3.3) and (3.4), then determine $n(\theta_i)$ and $m(\theta_i)$ from (3.9) and (3.11) respectively, such that

$$\begin{aligned} n(\theta_i) &= \min\left\{n > 0 : \sum_{i=1}^{n+1} s_i + \tau \geq R\left(\frac{\lambda_n + \omega\mu_n}{I_n + C_n}\right)\right\}, \\ m(\theta_i) &= \min\left\{n > 0 : n \geq n(\theta_i), \quad \delta_n \geq 0\right\}. \end{aligned}$$

(3) Compute $C(n, \theta_i)$ for all $n \leq n(\theta_i)$ and $C(m(\theta_i), \theta_i)$. Then evaluate

$$C(n^*(\theta_i), \theta_i) = \min\left[\min_{n \leq n(\theta_i)} C(n, \theta_i), C(m(\theta_i), \theta_i)\right].$$

(4) Move i to $i+1$, go back to step (2) and continue up to $i = k$.

(5) Finally, the global minimum is equal to

$$C(n^*, \theta^*) = \min_{n,\theta} C(n, \theta) = \min_i C(n^*(\theta_i), \theta_i).$$

The corresponding IRR policy $(s_1^*, s_2^*, \ldots, s_n^*, s_{r,n}^*; \theta^*)$ is approximately an optimal IRR policy. ■

Obviously, the algorithm is always finite. Therefore, we can find approximately an optimal IRR policy after a finite number of steps of searching.

Chapter 4

Numerical examples and sensitivity analysis with discussion

4.1 Weibull Distribution Case

In this section, we study an example of the Weibull distribution.

Suppose the distribution of $X_i \mid X_i > 0$ follows the Weibull distribution $W(a, b_i)$ with density

$$f_i(x) = \begin{cases} ab_i x^{a-1} e^{-b_i x^a} & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Hence for $x > 0$,

$$F_i(x) = 1 - e^{-b_i x^a},$$

and for $0 < x < 1$,

$$F_i^{-1}(x) = \left[\frac{\ln(1-x)}{-b_i}\right]^{\frac{1}{a}}.$$

In this example, the following parameter values are chosen as the standard values for comparison: $p = 0.95$, $p' = 0.95$, $a = 2$, $b_i = 0.49 + 0.01i$, $\lambda_i = 0.01 + 0.001i$, $\mu_i = 0.1 + 0.001i$, for $i = 1, 2, 3, \ldots$. Inspection cost and repair cost are assumed to be directly proportional to the time length of inspection and repair with rate $I = 50$ and $C = 30$ respectively. The expected time length of replacement is 2 and the replacement cost is 80. The penalty rate $g(\theta) = 100\left(\frac{1}{\theta^4} - 1\right)$. The lower bound of availability α is 0.9. It is easy to check that Assumptions 1, 2, 3, 4' and 5' hold for this example.

Next, we divide the interval of $\theta \in [0.9, 1)$ into 50 equal spaced subintervals so that $\theta_k = 0.9 + 0.002k$, $k = 0, 1, 2, \ldots, 49$. To demonstrate the existence of the local turning point $m(\theta_i)$ for any θ_i. We write a Fortran programme to calculate the average cost $C(n, \theta)$ from $n = 0$ to $n = 500$ for $\theta = 0.9, 0.945, 0.99$. We then plot the average cost $C(n, \theta)$ versus the number of inspections n. In Figures 1-3, each exhibits that $C(n, \theta)$ has a local turning point. Here, θ_i, $n(\theta_i)$, $m(\theta_i)$, $n^*(\theta_i)$, and $C(n^*(\theta_i), \theta_i)$ are listed as follows:

θ_i	$n(\theta_i)$	$m(\theta_i)$	$n^*(\theta_i)$	$C(n^*(\theta_i), \theta_i)$
0.9	1	43	43	18.6929
0.945	1	44	44	15.2761
0.99	1	43	43	18.7094

Note that in all cases, $n^*(\theta_i) = m(\theta_i)$.

In Figure 4, we plot the average cost versus different values of θ. The minimum average cost in Figure 4 is based on the optimal policy for a given θ. The graph looks like a convex curve, which agrees with our intuition that policies with too high or too low availability are not feasible. If the availability is too high, the inspections and the repairs (if any) will be too frequent and it yields a large amount of inspection cost and repair cost. On the other hand, if the availability is too low, the cost compensate for the penalty will become very high.

Given the standard values of the parameters, according to the finite algorithm, we have the following optimal IRR policy:

Optimal number of inspections : $n^* = 44$.

Optimal availability at inspection times : $\theta = 0.960$.

Optimal times: s_i^*,, $i = 1, 2, \ldots, 44$.

0.286	0.276	0.273	0.271	0.268	0.266	0.263	0.261	0.259	0.257
0.254	0.252	0.250	0.248	0.246	0.244	0.243	0.241	0.239	0.237
0.235	0.234	0.232	0.231	0.229	0.228	0.226	0.225	0.223	0.222
0.220	0.219	0.218	0.216	0.215	0.214	0.212	0.211	0.210	0.209
0.208	0.207	0.205	0.204						

Optimal time : $s_{r,n^*}^* = 0.203$.

Minimum average cost : $C(s_1^*, s_2^*, \ldots, s_{r,n^*}^*; \theta^*) = 14.968$.

The results are also tabulated in Tables 1-6. The parameter values are located in the first row. The availabilities , the minimum average cost and the number of inspections are calculated and reported in the second, the third and the fourth rows respectively. Then the optimal policies are followed. Moreover, Tables 1-6 give the minimum average cost and the optimal IRR policy for different parameter values.

4.2 Gamma Distribution Case

In this section, an example with the gamma distribution is studied. First of all, we assume that $X_i \mid X_i \sim F_i$, $i = 1, 2, \ldots$, where $F_i(x) = F((1+0.01(i-1))x)$, where F is the gamma distribution $\Gamma(\alpha, \beta)$ with density

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \qquad 0 \leq x < \infty, \ \ \alpha, \ \ \beta > 0.$$

The following parameters values are chosen as the standard values for sensitivity analysis in the later section: $\alpha = 5$, $\beta = 0.5$, $p = 0.9$, $p' = 0.9$, $\lambda_i = 0.2$, $\mu_i = 0.3$, $I_i = 5(1 + 0.02(i - 1))$, $C_i = 8(1 + 0.02(i - 1))$ for $i = 1, 2, \ldots$. The penalty rate $g(\theta) = 5\left(1 - \frac{\theta - 0.9}{0.1}\right)$, for $\theta \geq 0.9$. The lower bound of availability α is 0.9. The expected time length of replacement τ and the replacement cost R are 3 and 200 respectively. It is easy to show that Assumptions 1-5 hold.

Similar to the example of the Weibull distribution in Section 4.1, we apply the finite algorithm to find an approximate optimal policy in which the interval $\theta \in [0.9, 1)$ is divided into 50 equal spaced subintervals for searching. For θ =0.9, 0.945 and 0.99, we calculate the average cost $C(n, \theta)$ from $n = 0$ to $n = 500$ for verifying the existence of the local turning points. Figures 5-7 are similar to Figures 1-3 which plot the average cost versus number of inspections. Tables 7-14 give the minimum average cost and the optimal IRR policy for different parameters values. Again, θ_i, $n(\theta_i)$, $n^*(\theta_i)$ and $C(n^*(\theta_i), \theta_i)$ are tabulated below. Note here that in all cases, $n^*(\theta_i) = m(\theta_i)$.

θ_i	$n(\theta_i)$	$m(\theta_i)^*$	$n^*(\theta_i)$	$C(n^*(\theta_i), \theta_i)$
0.9	1	40	40	8.1679
0.945	1	40	40	6.3971
0.99	1	41	41	5.8472

In Figure 8, the minimum average cost for given θ_i is plotted against θ_i. The graph once again exhibits a convex shape. By applying the finite algorithm to the standard values of parameters, the optimal policy is given as the following:

Optimal number of inspections : $n^* = 41$.

Optimal availability at inspection times : $\theta = 0.980$.

Optimal times: s_i^*, $i = 1, 2, \ldots, 41$.

3.059	2.947	2.918	2.890	2.862	2.835	2.808	2.782	2.756	2.731
2.706	2.682	2.658	2.634	2.611	2.589	2.566	2.544	2.523	2.502
2.481	2.460	2.440	2.420	2.401	2.381	2.363	2.344	2.326	2.308
2.290	2.272	2.255	2.238	2.222	2.205	2.189	2.173	2.157	2.142
2.126									

Optimal time : $s_{r,n^*}^* = 2.111$

Minimum average cost : $C(s_1^*, s_2^*, \ldots, s_{r,n^*}^*; \theta^*) = 5.597$

The results are also tabulated in Tables 7-14. Furthermore, Tables 7-14 also give the minimum average cost and the optimal IRR policy for different parameter values.

4.3 Exponential Distribution Case

In this section, an geometric model with exponential distribution is studied. Assume that $X_i \mid X_i > 0 \sim F_i$, $i = 1, 2, \ldots$, where $F_i(x) = F(a^{i-1}x)$. F is exponential distribution $\exp(\gamma)$ with density

$$f_i(x) = \begin{cases} \gamma e^{-\gamma x} & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Hence for $x > 0$,

$$F_i(x) = F(a^{i-1}x) = 1 - e^{-\gamma a^{i-1} x},$$

and for $0 < x < 1$,

$$F_i^{-1}(x) = \frac{1}{a^{i-1}} F^{-1}(x) = \frac{1}{a^{i-1}} \cdot \frac{\ln(1-x)}{-\gamma}, \quad i = 1, 2, \ldots.$$

The following parameters values are chosen as the standard values for comparison: $\gamma = 1.0$, $a = 1.025$, $p = 0.9$, $p' = 0.9$, $I_i = 0.5$, $C_i = 1.5$ for $i = 1, 2, \ldots$. The penalty rate $g(\theta) = 100\left(\frac{1}{\theta^5} - 1\right)$. The lower bound of availability α is 0.9. The replacement cost R is 20. Time lengths of inspections, repairs (if any) and replacements are all assumed to be negligible. Obviously, Assumptions 1-5 are satisfied.

Note that this example is eventually a special case of the general IRR model in which inspection times, repair times and replacement times are assumed to be negligible. Moreover, it satisfies all the conditions of Lam's IRR model [3]. Therefore, the optimal policies obtained by using the general IRR model or Lam's IRR model [3] should be the same.

For the following three θ values, the key points, the local turning points, e.t.c. are calculated. The results are given below.

θ_i	$n(\theta_i)$	$m(\theta_i)^*$	$n^*(\theta_i)$	$C(n^*(\theta_i),\theta_i)$
0.9	1	37	37	89.6503
0.945	1	38	38	69.0133
0.99	1	39	39	201.078

Figures 9-11 plot the average cost $C(n,\theta)$ from $n = 0$ to $n = 500$ for θ =0.9, 0.945, 0.99. A local turning point appears in each figure. Figure 12 plots the availability against the minimum average cost. Just like Figures 4 and 8, once again it exhibits a convex shape. Applying the finite algorithm to the standard values of the parameters, the optimal policy is given as the following.

Optimal number of inspections : $n^* = 38$.

Optimal availability at inspection times : $\theta = 0.946$.

Optimal times: s_i^*, $i = 1, 2, \ldots, 38$.

5.55E-02	4.89E-02	4.77E-02	4.65E-02	4.54E-02	4.43E-02	4.32E-02
4.21E-02	4.11E-02	4.01E-02	3.91E-02	3.82E-02	3.73E-02	3.63E-02
3.55E-02	3.46E-02	3.37E-02	3.29E-02	3.21E-02	3.13E-02	3.06E-02
2.98E-02	2.91E-02	2.84E-02	2.77E-02	2.70E-02	2.64E-02	2.57E-02
2.51E-02	2.45E-02	2.39E-02	2.33E-02	2.27E-02	2.22E-02	2.16E-02
2.11E-02	2.06E-02	2.01E-02				

Optimal time : s_{r,n^*}^*=1.96E-02

Minimum average cost : $C(s_1^*, s_2^*, \ldots, s_{r,n^*}^*; \theta^*) = 68.9715$

Tables 15-20 show the optimal policies for different parameters values. Note that the optimal policy obtained here agrees with the optimal policy given by Lam's IRR model. This means that our IRR model is a generalization of Lam's IRR model.

4.4 Sensitivity analysis

Sensitivity analysis is to study the behavior of the optimal policy due to a change in the parameter values. In this section, we are going to analyze the behavior of an optimal IRR policy when one of the parameters in the model varies while the others are kept unchanged. In practice, the values of the parameters are unknown and have to be estimated. If some of the parameters in the model cannot be estimated accurately, the change in the values of the minimum average cost due to a change in the values of the parameters is of particular interest.

Now, suppose that only one parameter is estimated inaccurately, but the other parameters are correctly estimated. Based on the estimates of the parameters, an "optimal policy" is derived. This "optimal policy" is not really the true optimal policy, we can then call the "optimal policy" as the estimated policy. The corresponding availability is called the estimated availability. The corresponding "minimum average cost" is not the true minimum average cost and is called the estimated average cost. They are tabulated in the second and third rows of Tables 1-20. By applying an estimated policy to the true parameters, the average cost is the true average cost (but not the minimum). Obviously, the estimated policies based on the inaccurate estimates of parameters are different from the optimal policy based on the true parameters. Thus, the number of inspections, the availability and the inspection times may be quite different.

Our main concern is two-folded. Firstly, we may want to know whether this estimation error will cause a large amount of rise in average cost. If the average cost is much higher than the minimum average cost, the IRR policy itself will become vulnerable because just a minor error in parameter value will cause a huge amount of cost burden. If it is really the case, we should pay more attention on the estimation procedure of parameters in the model, and perhaps we should modify our model to overcome this shortcoming. Secondly, we may want to know if the estimated policy is applied to the true parameters, whether the availability will maintain high enough, still be greater than the preassigned lower bound α say.

Sensitivity analysis of the Weibull distribution example.

Assume that the standard values are taken as the true parameters, the availability at inspection times are 0.96 and the average cost is 14.968. In the study of the sensitivity study, we want to know whether an inaccurate parameter will cause a great amount rise in average cost and whether the availability will be less than the lower bound, 0.9 say.

In Tables 1-6, the true values of parameters and the true minimum average cost are marked by an "$*$". The estimated parameters are located in the first row. Based on the estimated parameters, estimated IRR polices are formulated.

When we apply the estimated IRR policy to the true parameters, the availabilities at inspection times will no longer be the same as that obtained by using the optimal policy. The estimated polices $\{s_1, s_2, \ldots; s_{r,n}\}$ and the true availabilities at inspection times are listed in the corresponding columns under the headings of s_i and θ. Finally, the true minimum availability of using an estimated policy to the true parameter, the true average cost and the efficiency are reported in the last three rows at the bottom of each table. Note that the efficiency of a policy is defined as the ratio of the minimum average cost to the average cost of using the policy.

For example, in Table 1, if a is inaccurately estimated as 1.5, the estimated policy takes 41 inspections with s_i, $i = 1, 2, \ldots, 41$ as 0.237, 0.226, $\ldots$, 0.155, and $s_{r,n} = 0.154$. By using the estimated policy to the true parameter value $a = 2$, the availabilities at s_i are 0.972, 0.973, ..., in which the true minimum value of availability is 0.972. The true average cost (but not the minimum) is 15.870. Since the minimum average cost for $a = 2$ is 14.968, the efficiency of the estimated policy is $14.968/15.870 = 0.943$. The sensitivity analysis for the other parameters can be carried out in a similar way.

The following are the true parameters and the maximum percentage error in Tables 1-6.

Table	parameter	true parameter value	maximum percentage error
1	a	2	50%
2	b_i	$0.49 + 10^{-4}i$	50%
3	p	0.95	4.2%
4	p'	0.95	4.2%
5	λ_i	$0.01 + 10^{-3}i$	50%
6	μ_i	$40.1 + 10^{-3}i$	50%

We only study the sensitivity analysis for the above six parameters because it is difficult to estimate their exact values in practice. From the results tabulated in Tables 1-6, we can find that, in most cases, the efficiencies are higher than 0.95 even if the errors of a parameter are over 50%. For parameters b_i, the efficiencies are all higher than 0.92. The average cost seems to be a little more sensitive to the parameter a than others. If the parameter a is inaccurately estimated as 1, the true average cost due to the use of the estimated policy is 18.917, it has increased by 26.4%, and the efficiency of the estimated policy is 0.791. In this case, the percentage error of parameter is 50%. On the other hand, all policies in Tables 1-6 have the availabilities not less than the lower bound 0.9.

Sensitivity analysis of the gamma distribution example.

Assume that the standard values of the parameters are taken as the true parameters. Then the optimal policy takes 41 inspections. The availabilities at inspection times are kept to higher than 0.98. The minimum average cost is 5.597.

The sensitivity analysis here is similar to that of the Weibull distribution example. The parameters under study are listed as follows.

Table	parameter	true parameter value	maximum percentage of error
7	α	5	40%
8	β	0.5	40%
9	p	0.9	10%
10	p'	0.9	10%
11	λ_i	0.2	50%
12	μ_i	0.3	50%
13	I_i	5(1+0.02(i-1))	50%
14	C_i	8(1+0.02(i-1))	50%

From the results reported in Tables 7-14, we can see that, in most cases, the efficiencies are higher than 0.96 even if the percentage errors are 50%. However, the average costs are comparatively sensitive to the parameters values of the distribution F. If the parameter β varies, the efficiencies of the average cost is higher than 0.73. For the parameter α, the efficiencies of the average cost are higher than 0.7 only. All policies in Tables 7-14 have the availabilities not less than the lower bound 0.9.

Sensitivity analysis of the Exponential distribution example.

The standard values of the parameters are taken as the true parameters. By using the finite algorithm, we obtain an optimal policy which takes 38 inspections. The availability is 0.946 while the average cost is 68.9715. The following parameters are under study:

Table	parameter	true parameter value	maximum percentage of error
15	γ	1	50%
16	a	1.025	2.43%
17	p	0.9	11.1%
18	p'	0.9	11.1%
19	I_i	0.01	50%
20	C_i	0.5	50%

From the results in Tables 17-20, the average cost is insensitive to the change of the values of p, p', I_i and C_i in which the efficiencies are all higher than 0.94. If the parameter γ is inaccurately estimated as 0.5, the efficiency of the estimated policy is 0.906. However, the average cost is sensitive to the change of the geometric ratio a. If a is underestimated as 1.013, the efficiency of the estimated policy downfalls to 0.73. Moreover, the minimum availability drops to 0.909.

Conclusion of sensitivity analysis

From the studies of three Examples above, in most cases, we can see that the true availabilities are not below the preassigned lower bound α. Moreover, the average cost is insensitive to the values of p, p', λ_i, μ_i, I_i and C_i. This is an advantage of our model. However, the average cost seems to be slightly sensitive to the parameters value of the distribution of the system. In the sensitivity analysis in the Exponential distribution case, it even happens that the general IRR policy can not afford a minor error of the ratio a. It is not surprising because only a minor change of a, the distribution of X_i will varies a lot. (Fortunately, in modelling a data set which involves a monotone trend, Lam and Zhu [4] have introduced an estimator $\hat{a}$ such that $\hat{a} - a$ is asymptotically normal with mean 0 and variance of order $O(n^{-3})$.) Thus, when the general IRR model is put into application, we should take great care of the estimation of the parameter in the distribution F.

4.5 Conclusion and Further development

Over and above, there are at least three advantages of the general IRR model. Firstly, it is a maintenance policy for a deteriorating system in which the successive conditional lifetime distributions are stochastically decreasing. Secondly, the general IRR model always pays special attention to the system availability. Thirdly, our general IRR model helps to control the cost at the lowest level. Eventually, the finite algorithm is quite simple and can be performed easily.

Nevertheless, there is some space for further research. These aspect include:

1. For IRR model studied in Lam [3], the availabilities θ_i can take different values at different inspection times. Can we generalize our IRR model to this case ?

2. The general IRR model highly depends on the estimation of the parameter values. After each inspection, we may have more information for estimation of the system parameters. Can we develop an adaptive IRR model for using the updated estimates after each inspection ?

3. The aim of the installation of a standby system is to improve the reliability and the availability of a production process. Can we calculate the reliability of the production process ? Furthermore, as Thomas [4] had studied, can we develop a IRR model so that the expected time until a catastrophe occurs is maximized ?

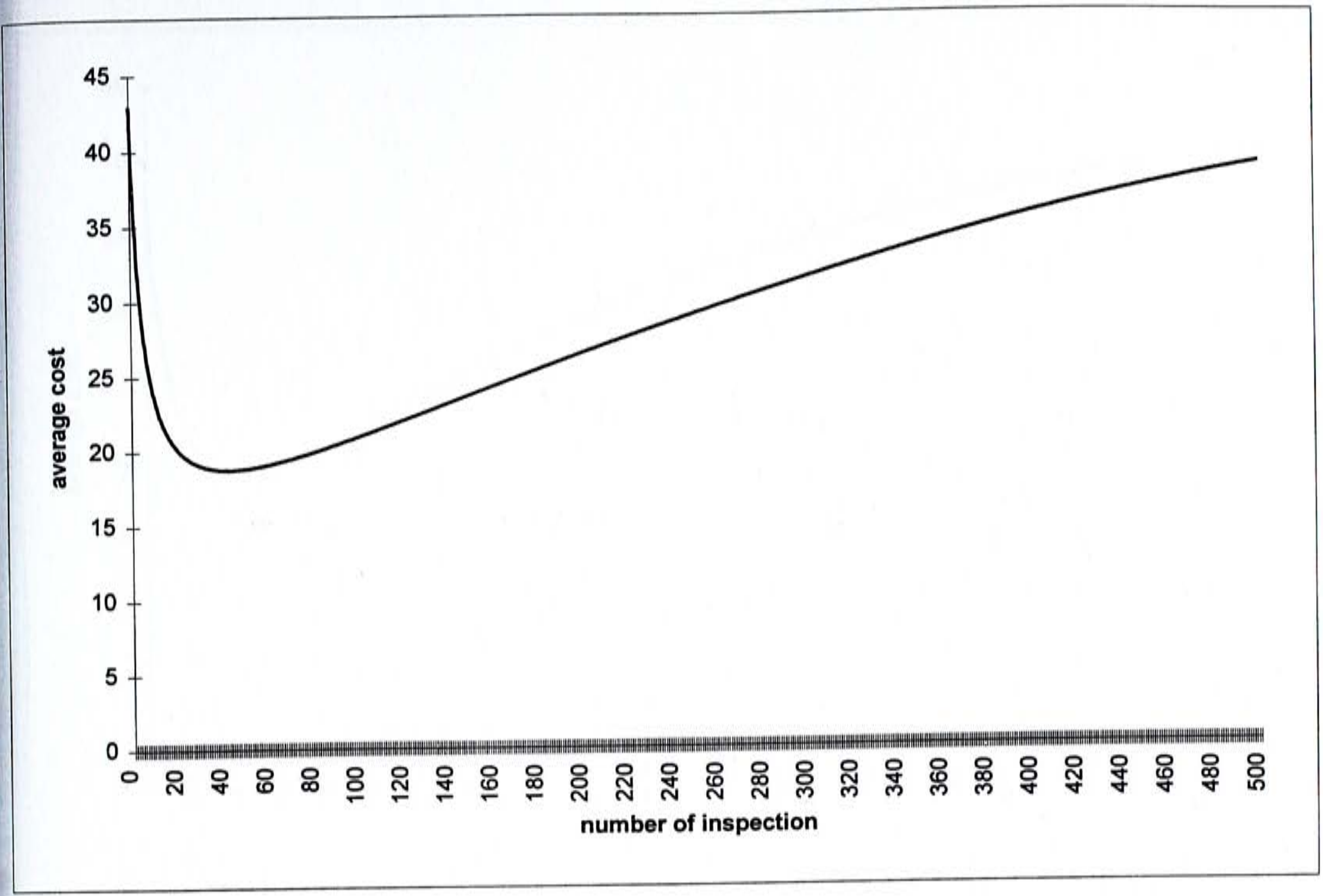


Figure 1. *Number of inspections versus average cost, given θ = 0.9 for $W(2,0.49+0.01i)$.*

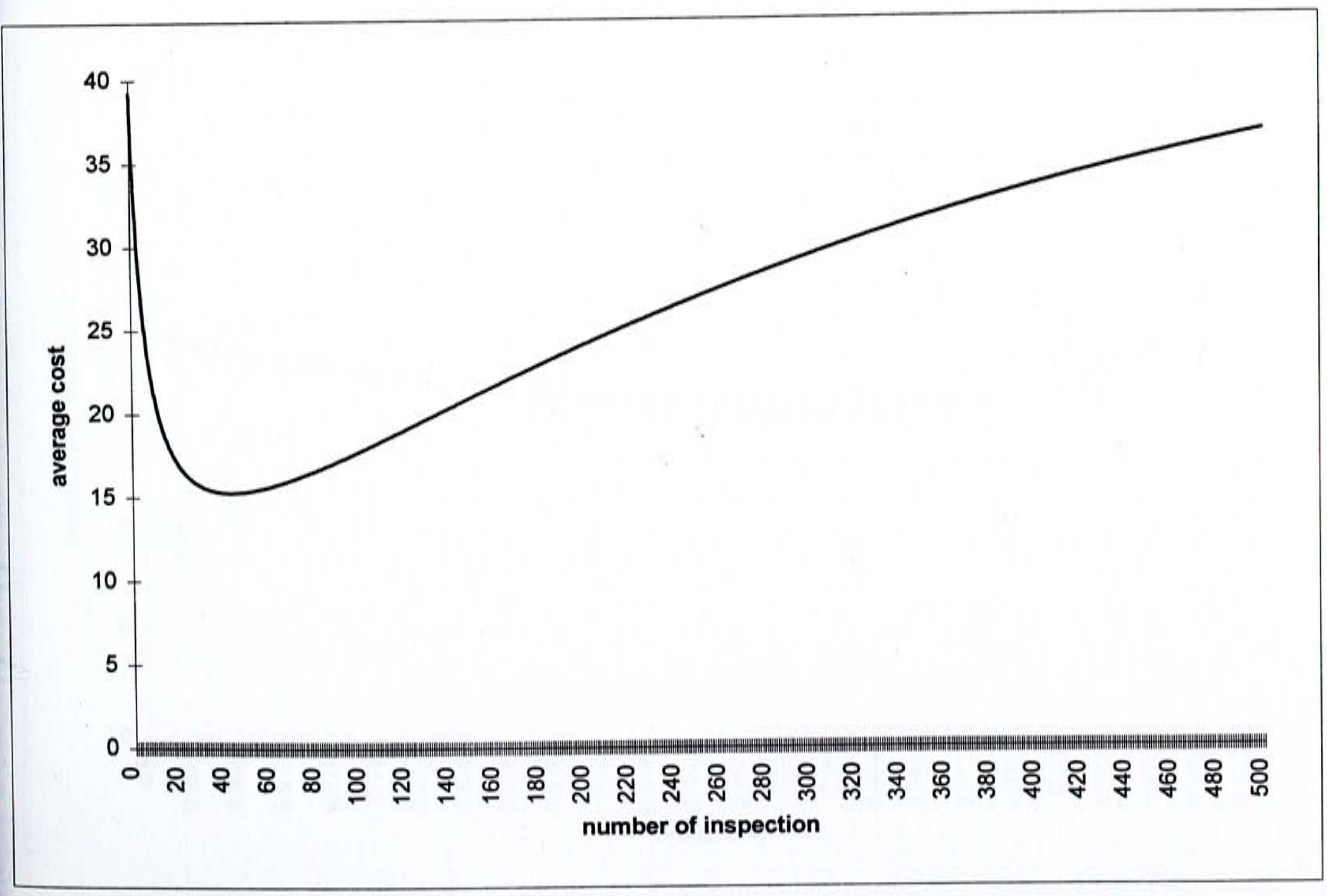


Figure 2. *Number of inspections versus average cost, given θ = 0.945 for $W(2,0.49+0.01i)$.*

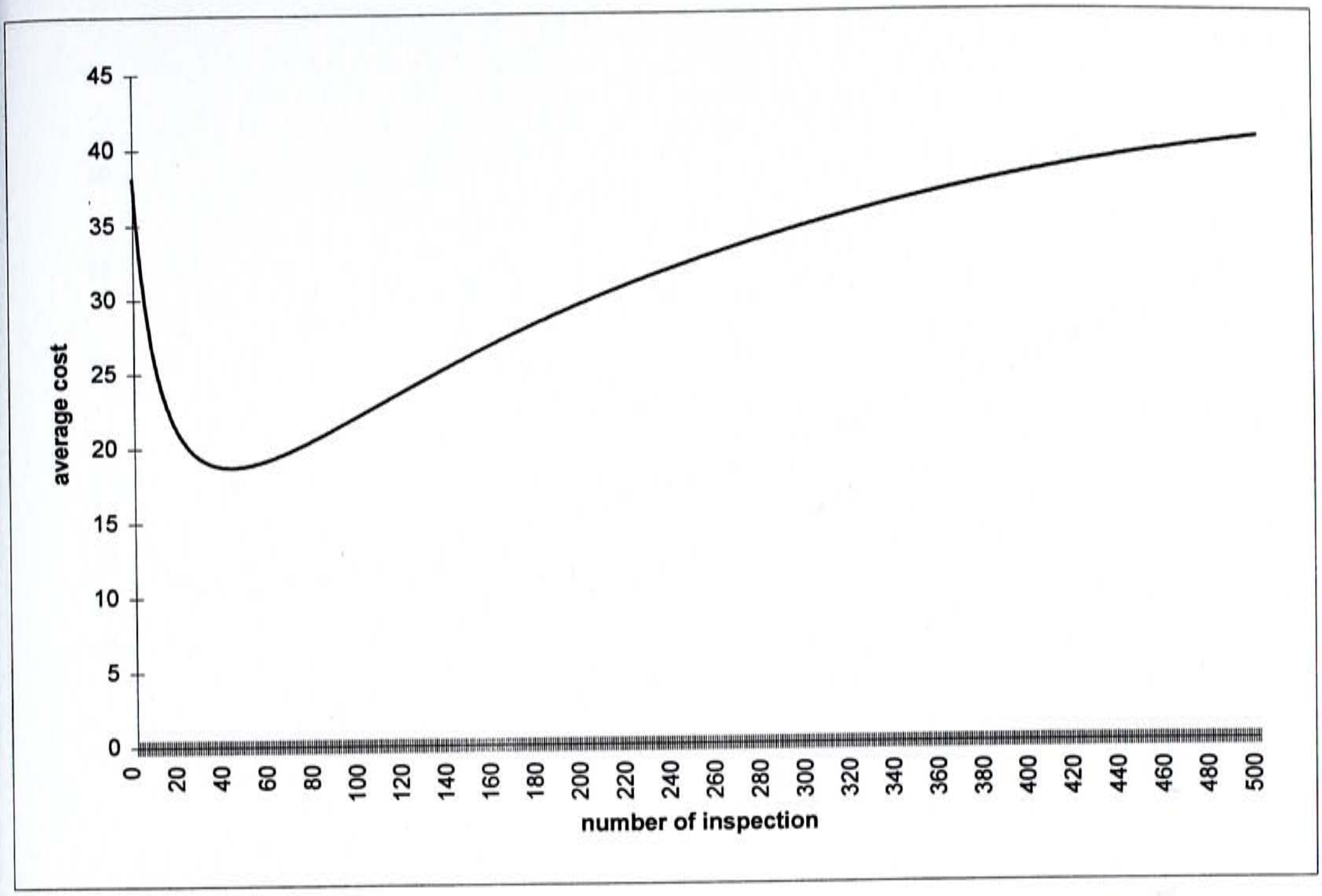


Figure 3. *Number of inspections versus average cost, given* θ *= 0.99 for* $W(2, 0.49+0.01i)$.

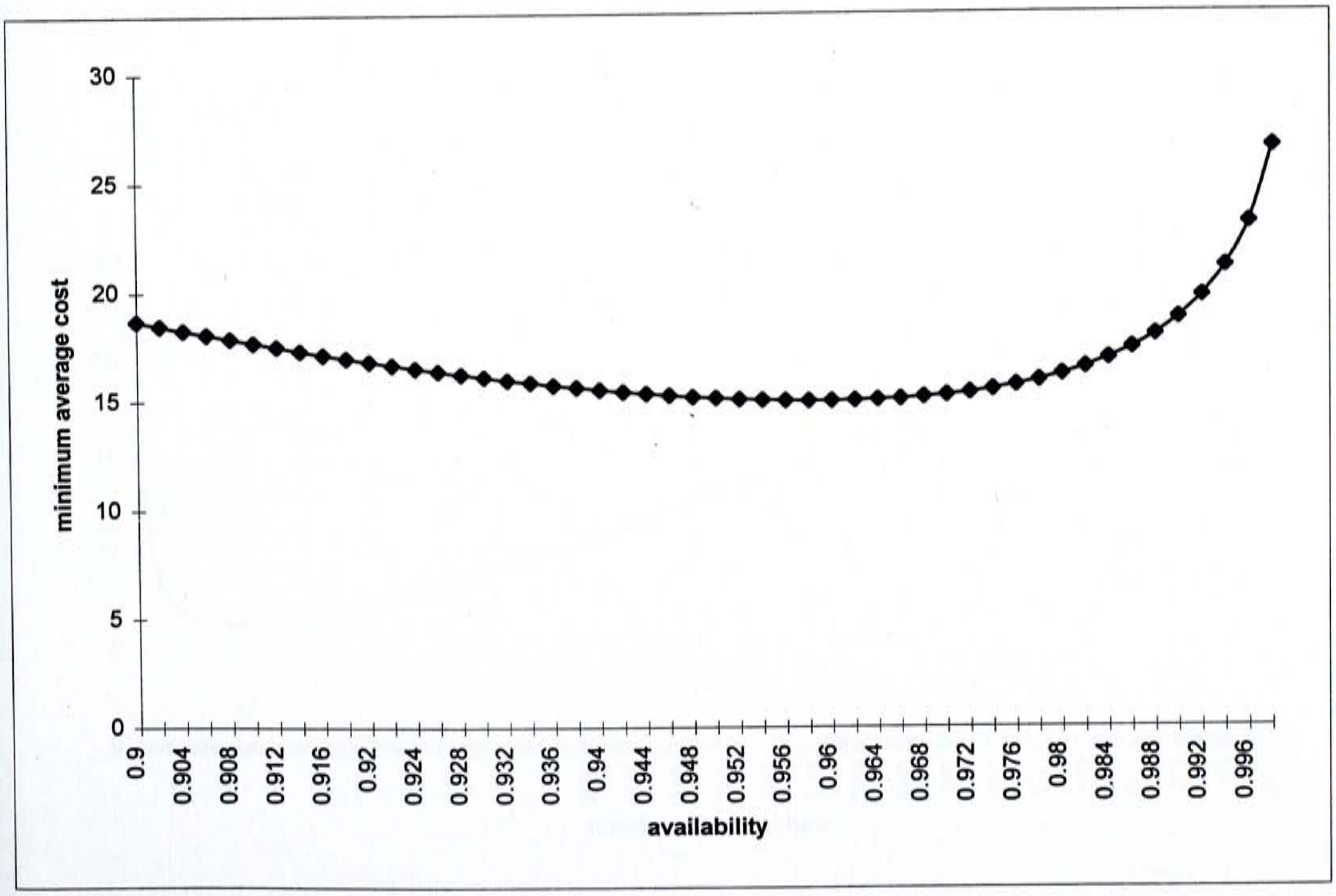


Figure 4. *Availability versus minimium average cost for* $W(2, 0.49+0.01i)$.

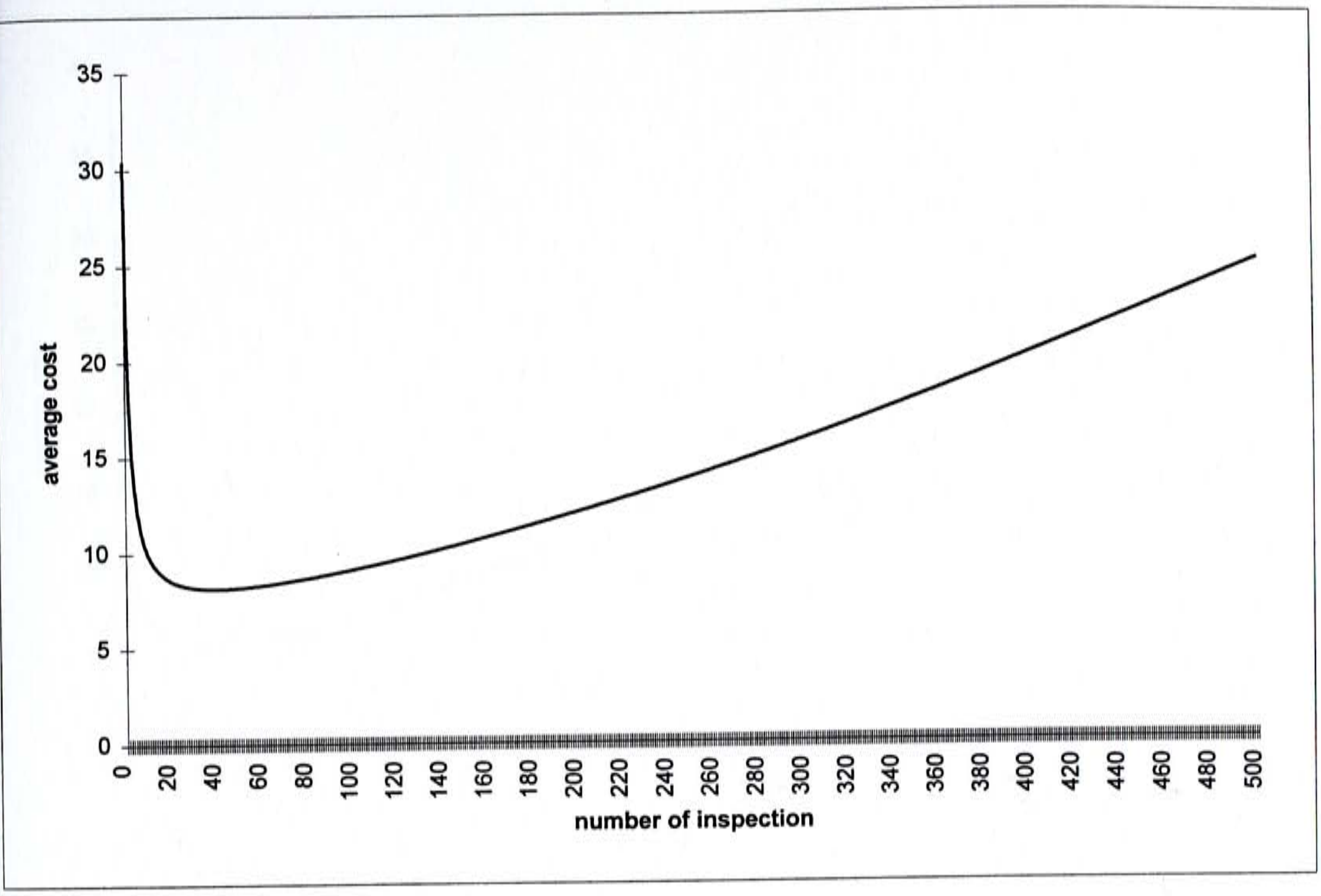


Figure 5. *Number of inspections versus average cost, given* $\theta = 0.9$ *for* $\Gamma(5,0.5)$.

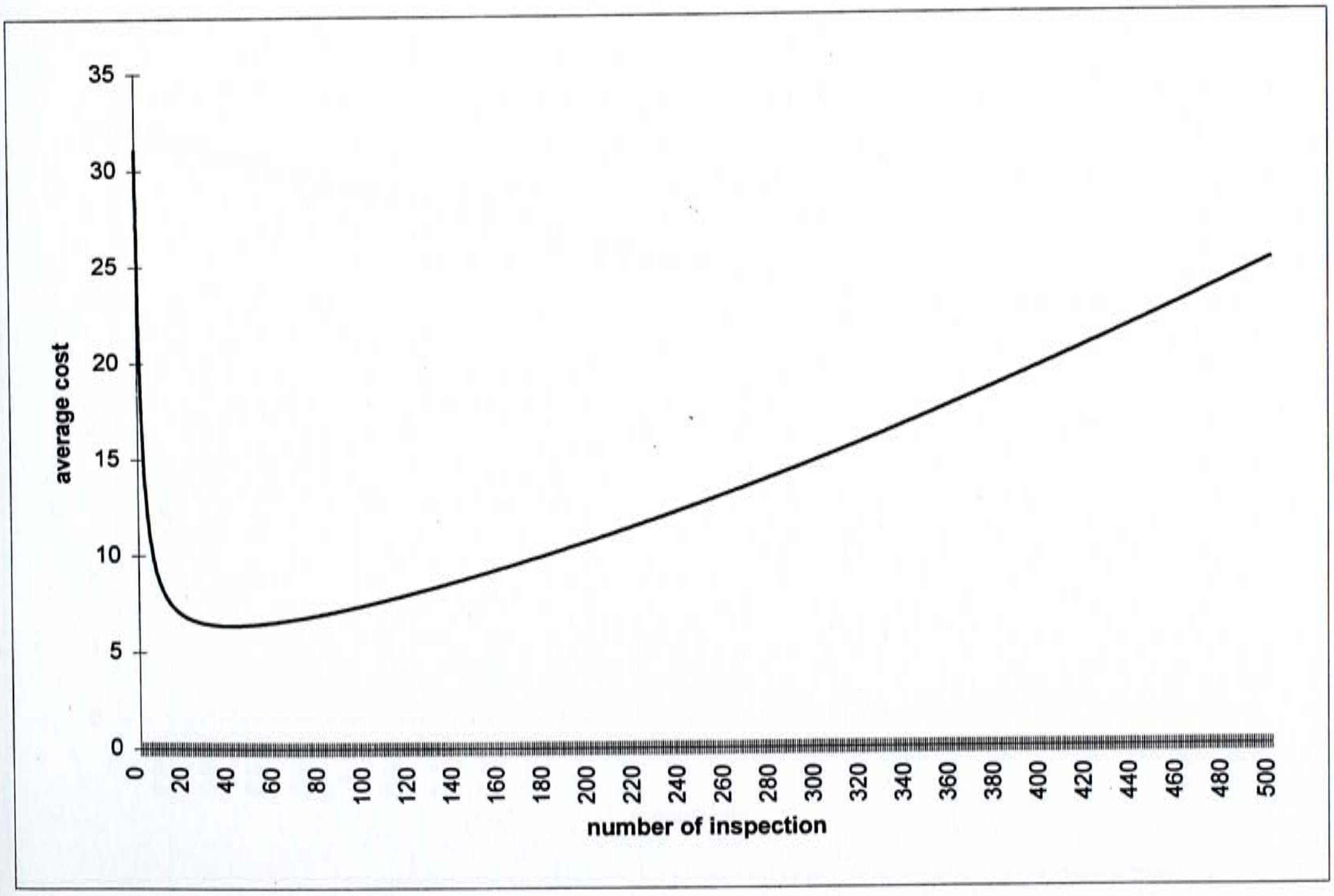


Figure 6. *Number of inspections versus average cost, given* $\theta = 0.945$ *for* $\Gamma(5,0.5)$.

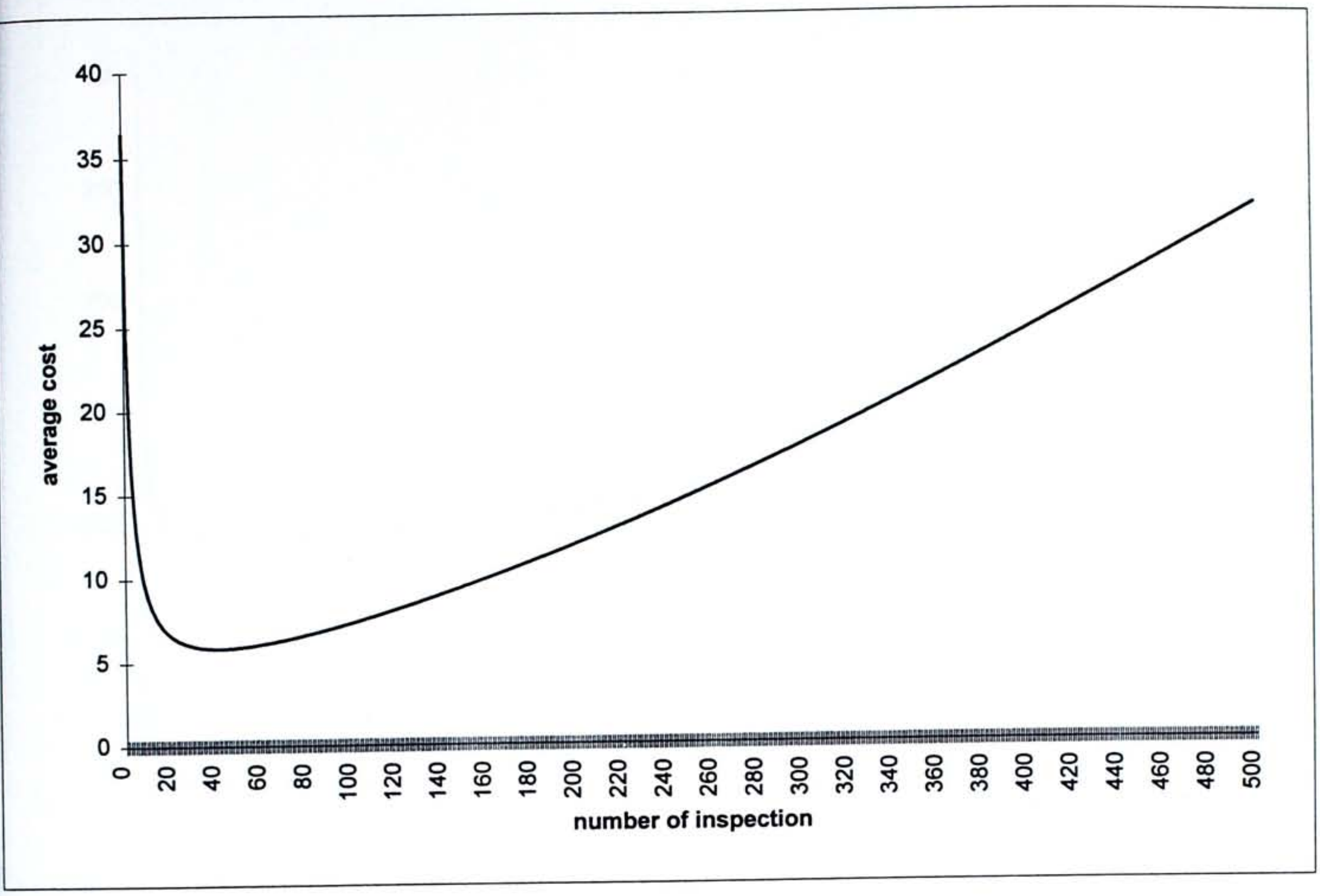


Figure 7. *Number of inspections versus average cost, given* $\theta = 0.99$ *for* $\Gamma(5,0.5)$.

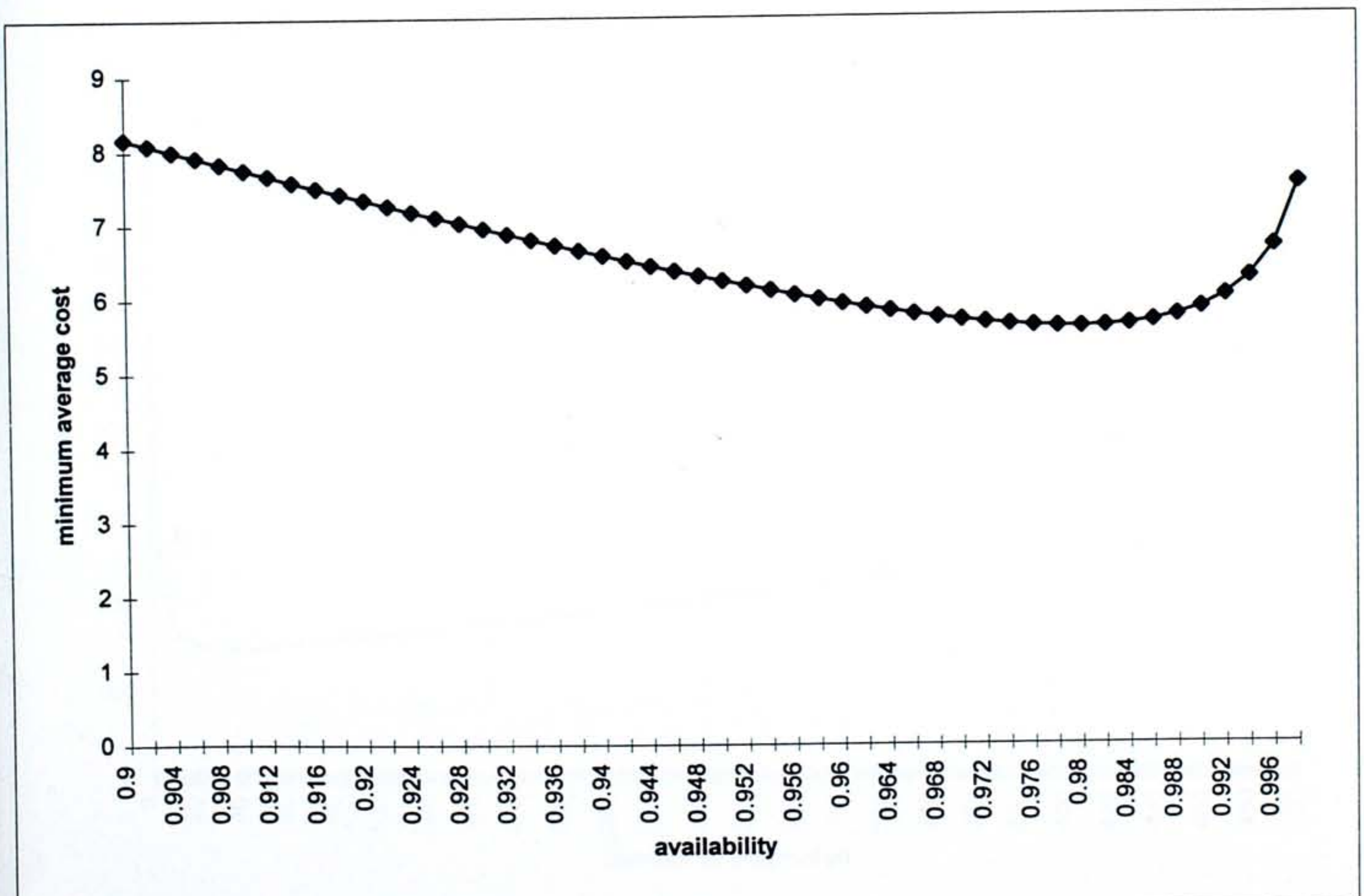


Figure 8. *Availability versus minimium average cost for* $\Gamma(5,0.5)$.

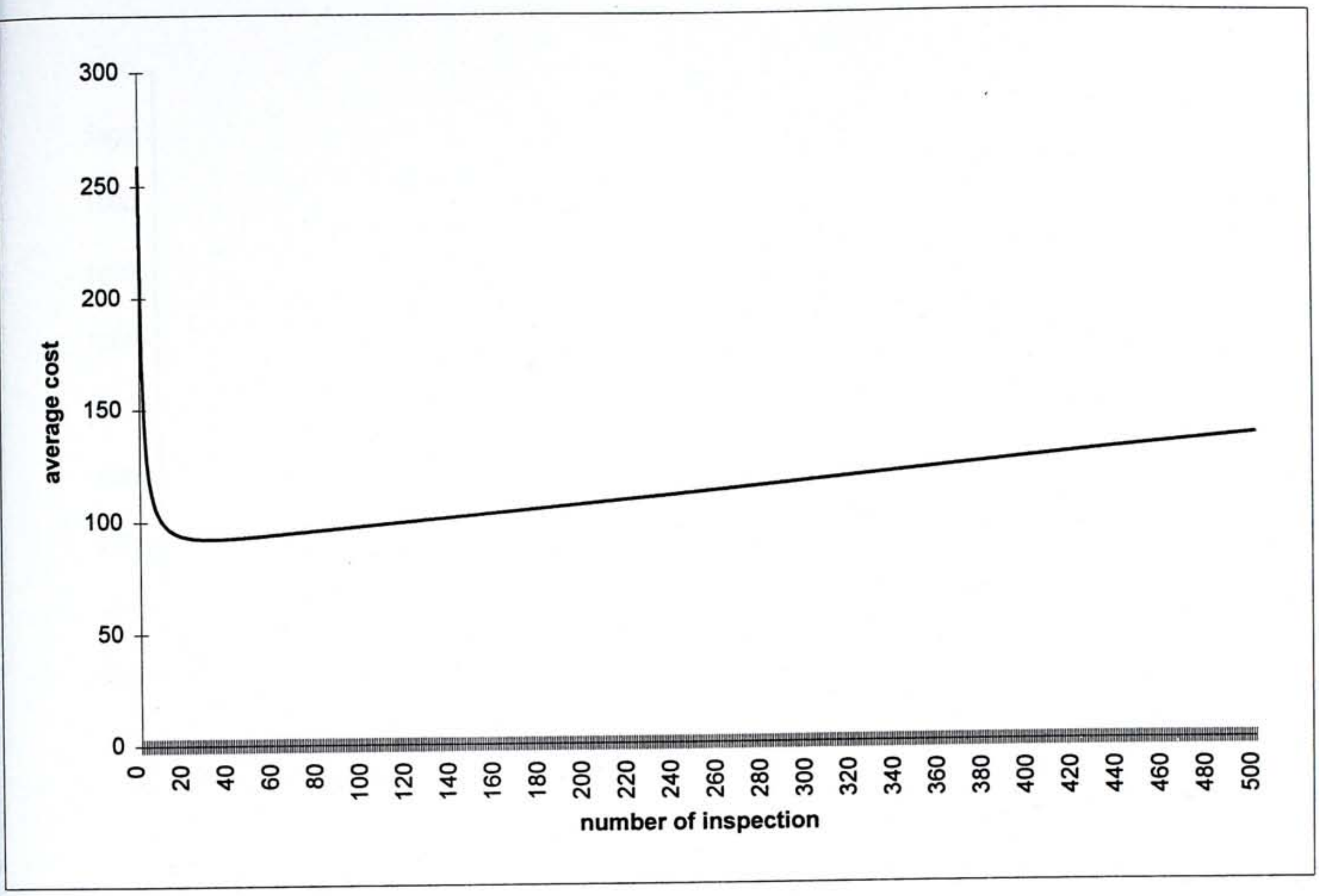


Figure 9. *Number of inspections versus average cost, given θ=0.9 for exp(1).*

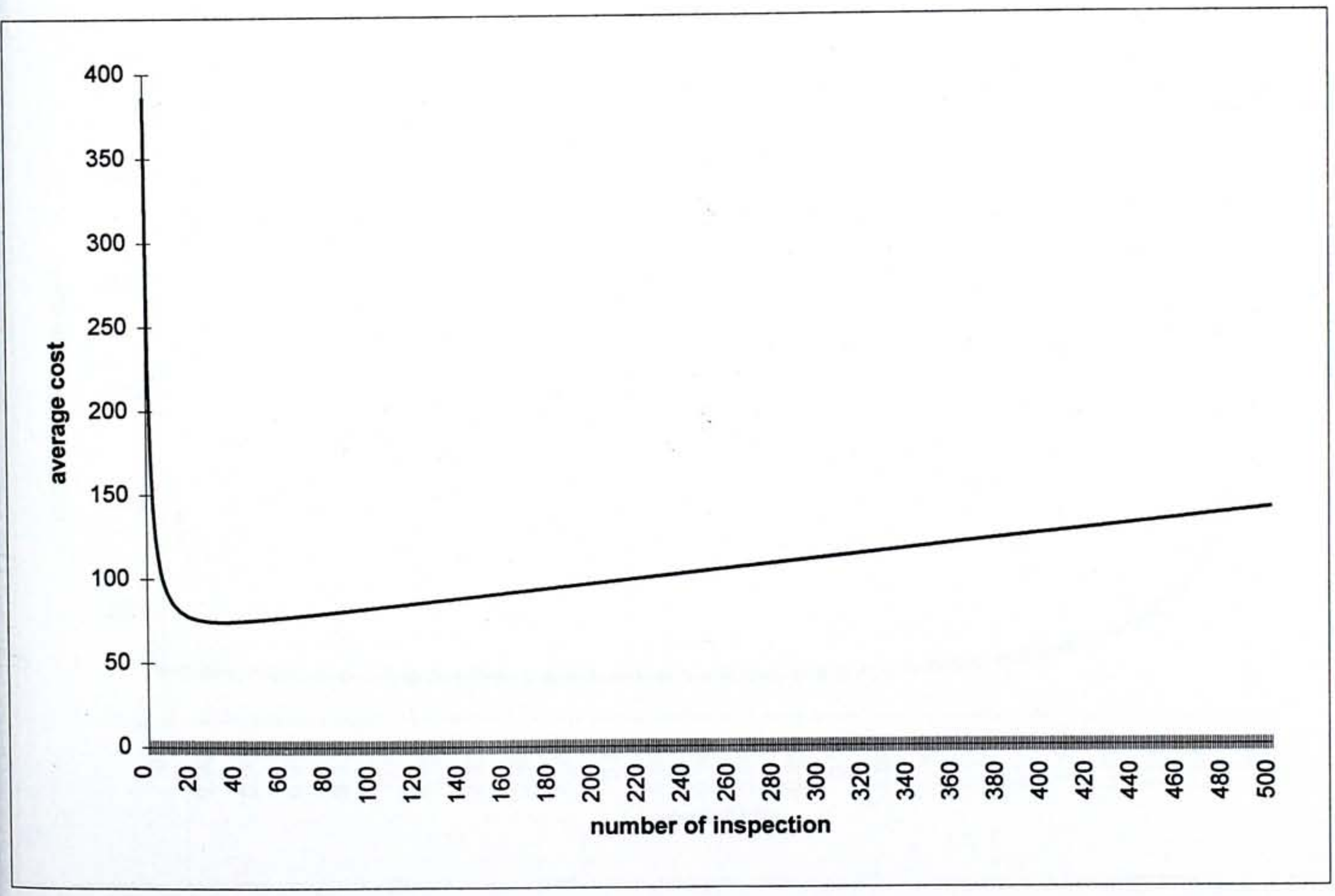


Figure 10. *Number of inspections versus average cost, given θ=0.945 for exp(1).*

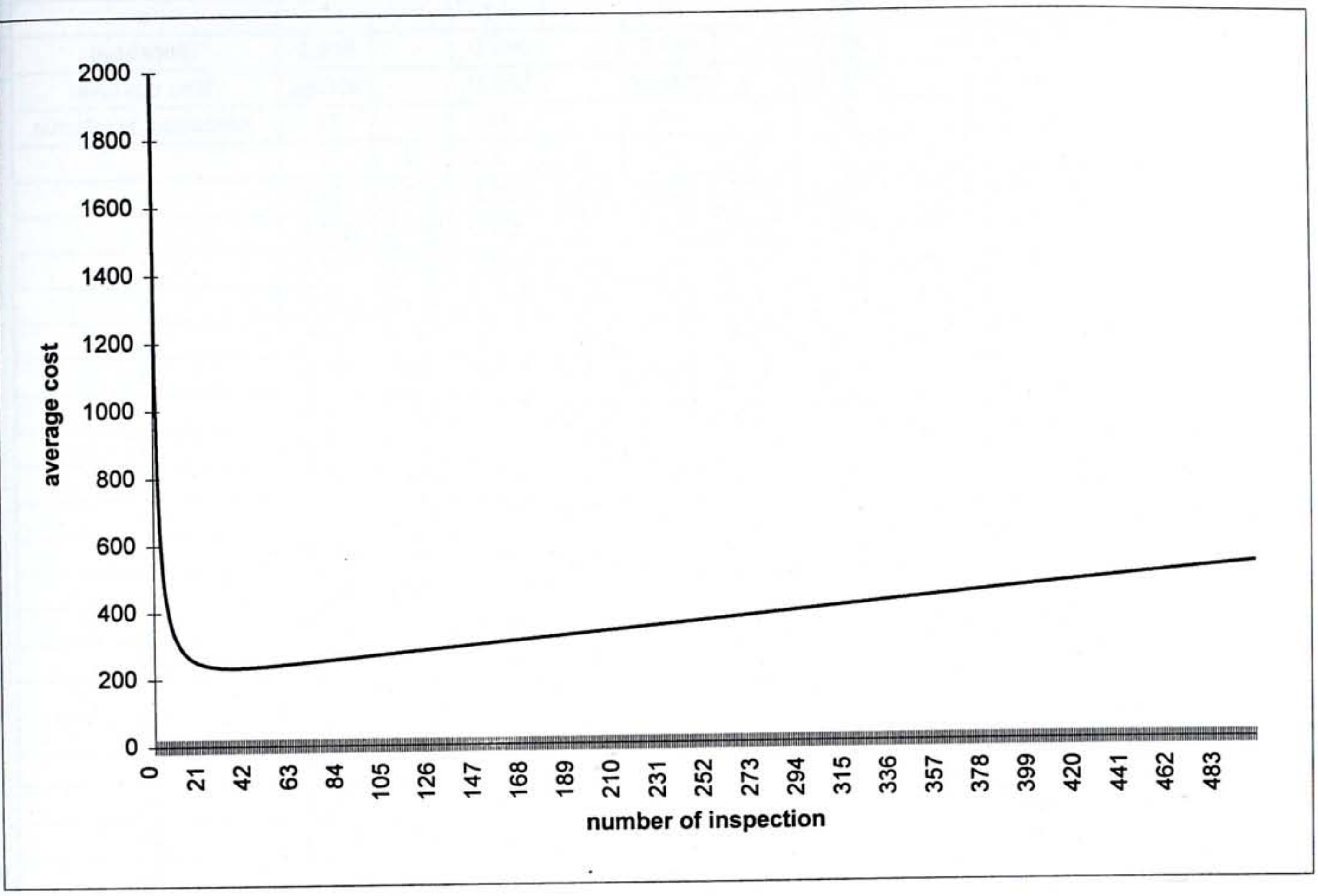


Figure 11. *Number of inspections versus average cost, given θ=0.99 for exp(1).*

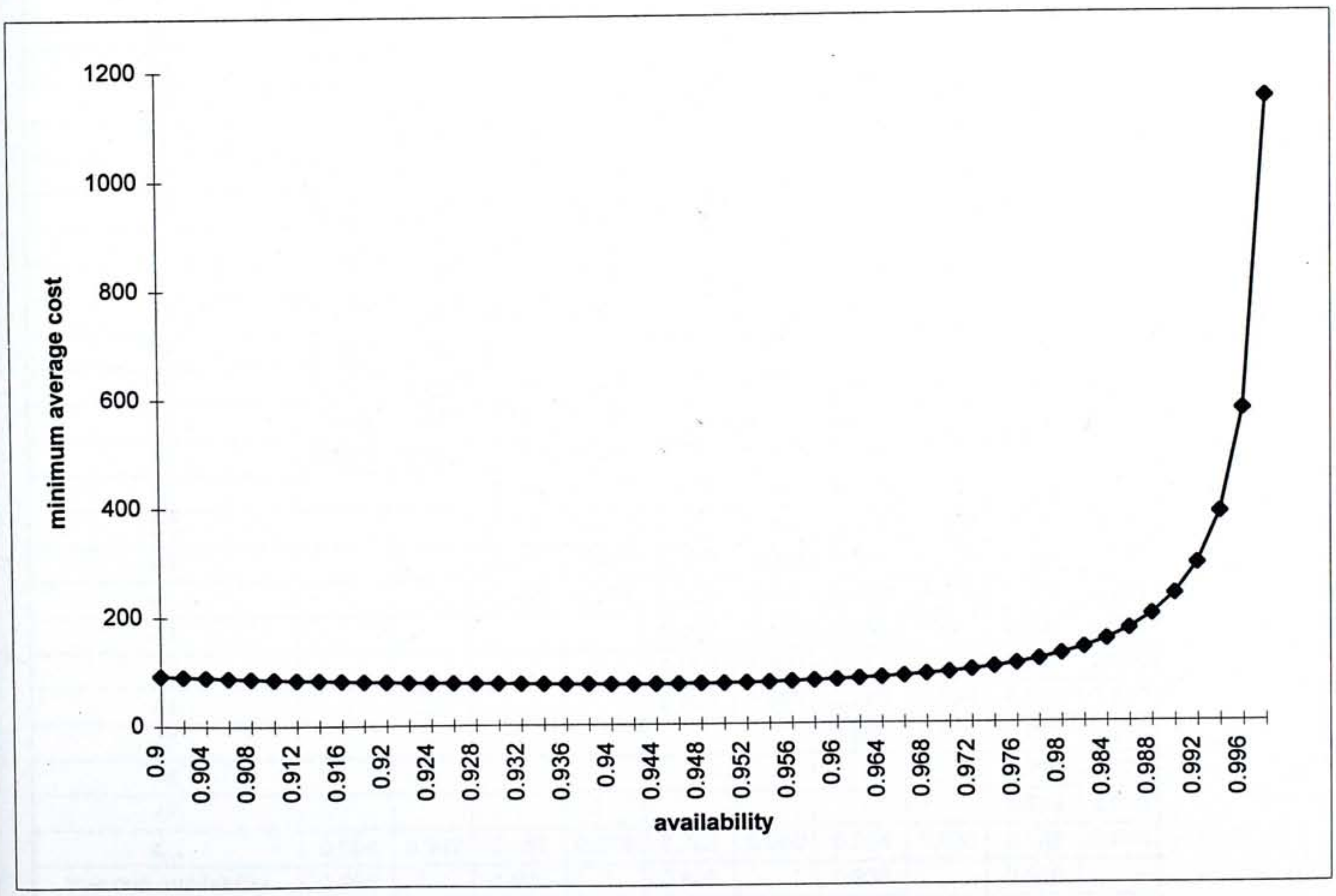


Table 12. *Availability versus minimum average cost for exp(1).*

Table 1. *The minimum average cost and the optimal IRR policy as a varies for W(a,b_i).*

a	1		1.5		2*		2.5		3	
availability	0.918		0.944		0.960		0.968		0.976	
average cost	26.308		18.895		14.968*		12.576		10.978	
number of inspection	37		41		44		45		47	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	0.171	0.985	0.237	0.972	0.286	0.960	0.335	0.945	0.365	0.936
2	0.160	0.986	0.226	0.973	0.276	0.960	0.326	0.945	0.356	0.934
3	0.157	0.987	0.223	0.973	0.273	0.960	0.323	0.944	0.354	0.934
4	0.154	0.987	0.220	0.973	0.271	0.960	0.321	0.944	0.352	0.933
5	0.151	0.987	0.218	0.973	0.268	0.960	0.319	0.944	0.350	0.933
6	0.148	0.987	0.215	0.974	0.266	0.960	0.316	0.944	0.348	0.933
7	0.145	0.988	0.212	0.974	0.263	0.960	0.314	0.944	0.345	0.932
8	0.143	0.988	0.210	0.974	0.261	0.960	0.312	0.943	0.343	0.932
9	0.140	0.988	0.208	0.974	0.259	0.960	0.310	0.943	0.341	0.931
10	0.138	0.988	0.205	0.974	0.257	0.960	0.307	0.943	0.340	0.931
11	0.136	0.988	0.203	0.974	0.254	0.960	0.305	0.943	0.338	0.931
12	0.134	0.989	0.201	0.974	0.252	0.960	0.303	0.943	0.336	0.930
13	0.131	0.989	0.198	0.975	0.250	0.960	0.301	0.943	0.334	0.930
14	0.129	0.989	0.196	0.975	0.248	0.960	0.299	0.942	0.332	0.930
15	0.127	0.989	0.194	0.975	0.246	0.960	0.298	0.942	0.330	0.929
16	0.125	0.989	0.192	0.975	0.244	0.960	0.296	0.942	0.329	0.929
17	0.123	0.989	0.190	0.975	0.243	0.960	0.294	0.942	0.327	0.929
18	0.122	0.990	0.188	0.975	0.241	0.960	0.292	0.942	0.325	0.928
19	0.120	0.990	0.187	0.975	0.239	0.960	0.290	0.941	0.324	0.928
20	0.118	0.990	0.185	0.976	0.237	0.960	0.289	0.941	0.322	0.927
21	0.116	0.990	0.183	0.976	0.235	0.960	0.287	0.941	0.321	0.927
22	0.115	0.990	0.181	0.976	0.234	0.960	0.285	0.941	0.319	0.927
23	0.113	0.990	0.180	0.976	0.232	0.960	0.284	0.941	0.318	0.926
24	0.112	0.990	0.178	0.976	0.231	0.960	0.282	0.941	0.316	0.926
25	0.110	0.991	0.176	0.976	0.229	0.960	0.281	0.941	0.315	0.926
26	0.109	0.991	0.175	0.976	0.228	0.960	0.279	0.940	0.313	0.926
27	0.107	0.991	0.173	0.976	0.226	0.960	0.278	0.940	0.312	0.925
28	0.106	0.991	0.172	0.976	0.225	0.960	0.276	0.940	0.311	0.925
29	0.104	0.991	0.170	0.976	0.223	0.960	0.275	0.940	0.309	0.925
30	0.103	0.991	0.169	0.977	0.222	0.960	0.274	0.940	0.308	0.924
31	0.102	0.991	0.167	0.977	0.220	0.960	0.272	0.940	0.307	0.924
32	0.101	0.991	0.166	0.977	0.219	0.960	0.271	0.939	0.305	0.924
33	0.099	0.992	0.165	0.977	0.218	0.960	0.270	0.939	0.304	0.923
34	0.098	0.992	0.163	0.977	0.216	0.960	0.268	0.939	0.303	0.923
35	0.097	0.992	0.162	0.977	0.215	0.960	0.267	0.939	0.302	0.923
36	0.096	0.992	0.161	0.977	0.214	0.960	0.266	0.939	0.301	0.922
37	0.095	0.992	0.160	0.977	0.212	0.960	0.264	0.939	0.299	0.922
38			0.158	0.977	0.211	0.960	0.263	0.939	0.298	0.922
39			0.157	0.977	0.210	0.960	0.262	0.938	0.297	0.922
40			0.156	0.977	0.209	0.960	0.261	0.938	0.296	0.921
41			0.155	0.978	0.208	0.960	0.260	0.938	0.295	0.921
42					0.207	0.960	0.259	0.938	0.294	0.921
43					0.205	0.960	0.257	0.938	0.293	0.920
44					0.204	0.960	0.256	0.938	0.292	0.920
45							0.255	0.938	0.291	0.920
46									0.290	0.920
47									0.289	0.919
$s_{r,n}$	0.094	0.992	0.154	0.978	0.203	0.960	0.254	0.938	0.288	0.919
true min availability	0.985		0.972		0.960		0.938		0.919	
true average cost	18.917		15.870		14.968		15.859		17.281	
efficiency	0.791		0.943		1.000		0.944		0.866	

Table 2. The minimum average cost and the optimal IRR policy as b_i varies for $W(a,b_i)$.

b_i	$0.245+5\times10^{-3}i$		$0.3675+7.5\times10^{-3}i$		$0.49+10^{-3}i$*		$0.6125+1.25\times10^{-2}i$		$0.735+1.5\times10^{-2}i$	
availability	0.964		0.962		0.960		0.958		0.956	
average cost	12.283		13.806		14.9683*		15.916		16.721	
number of inspection	44		44		44		43		43	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	0.383	0.929	0.321	0.950	0.286	0.960	0.262	0.966	0.245	0.970
2	0.370	0.929	0.310	0.950	0.276	0.960	0.253	0.966	0.237	0.970
3	0.366	0.929	0.307	0.950	0.273	0.960	0.251	0.966	0.234	0.970
4	0.363	0.929	0.304	0.950	0.271	0.960	0.248	0.966	0.232	0.970
5	0.359	0.929	0.302	0.950	0.268	0.960	0.246	0.966	0.230	0.970
6	0.356	0.929	0.299	0.950	0.266	0.960	0.244	0.966	0.228	0.970
7	0.353	0.929	0.296	0.950	0.263	0.960	0.241	0.966	0.226	0.970
8	0.350	0.929	0.294	0.950	0.261	0.960	0.239	0.966	0.224	0.970
9	0.347	0.929	0.291	0.950	0.259	0.960	0.237	0.966	0.222	0.970
10	0.344	0.929	0.289	0.950	0.257	0.960	0.235	0.966	0.220	0.970
11	0.341	0.929	0.286	0.950	0.254	0.960	0.233	0.966	0.218	0.970
12	0.338	0.929	0.284	0.950	0.252	0.960	0.231	0.966	0.216	0.970
13	0.335	0.929	0.281	0.950	0.250	0.960	0.229	0.966	0.215	0.970
14	0.333	0.929	0.279	0.950	0.248	0.960	0.228	0.966	0.213	0.970
15	0.330	0.929	0.277	0.950	0.246	0.960	0.226	0.966	0.211	0.970
16	0.328	0.929	0.275	0.950	0.244	0.960	0.224	0.966	0.210	0.970
17	0.325	0.929	0.273	0.950	0.243	0.960	0.222	0.966	0.208	0.970
18	0.323	0.929	0.271	0.950	0.241	0.960	0.221	0.966	0.206	0.970
19	0.320	0.929	0.269	0.950	0.239	0.960	0.219	0.966	0.205	0.970
20	0.318	0.929	0.267	0.950	0.237	0.960	0.218	0.966	0.203	0.970
21	0.316	0.929	0.265	0.950	0.235	0.960	0.216	0.966	0.202	0.970
22	0.313	0.929	0.263	0.950	0.234	0.960	0.214	0.966	0.200	0.970
23	0.311	0.929	0.261	0.950	0.232	0.960	0.213	0.966	0.199	0.970
24	0.309	0.929	0.259	0.950	0.231	0.960	0.211	0.966	0.198	0.970
25	0.307	0.929	0.258	0.950	0.229	0.960	0.210	0.966	0.196	0.970
26	0.305	0.929	0.256	0.950	0.228	0.960	0.209	0.966	0.195	0.970
27	0.303	0.929	0.254	0.950	0.226	0.960	0.207	0.966	0.194	0.970
28	0.301	0.929	0.253	0.950	0.225	0.960	0.206	0.966	0.192	0.970
29	0.299	0.929	0.251	0.950	0.223	0.960	0.205	0.966	0.191	0.970
30	0.297	0.929	0.249	0.950	0.222	0.960	0.203	0.966	0.190	0.970
31	0.295	0.929	0.248	0.950	0.220	0.960	0.202	0.966	0.189	0.970
32	0.293	0.929	0.246	0.950	0.219	0.960	0.201	0.966	0.188	0.970
33	0.292	0.929	0.245	0.950	0.218	0.960	0.200	0.966	0.187	0.970
34	0.290	0.929	0.243	0.950	0.216	0.960	0.198	0.966	0.185	0.970
35	0.288	0.929	0.242	0.950	0.215	0.960	0.197	0.966	0.184	0.970
36	0.286	0.929	0.240	0.950	0.214	0.960	0.196	0.966	0.183	0.970
37	0.285	0.929	0.239	0.950	0.212	0.960	0.195	0.966	0.182	0.970
38	0.283	0.929	0.238	0.950	0.211	0.960	0.194	0.966	0.181	0.970
39	0.281	0.929	0.236	0.950	0.210	0.960	0.193	0.966	0.180	0.970
40	0.280	0.929	0.235	0.950	0.209	0.960	0.192	0.966	0.179	0.970
41	0.278	0.929	0.234	0.950	0.208	0.960	0.190	0.966	0.178	0.970
42	0.277	0.929	0.232	0.950	0.207	0.960	0.189	0.966	0.177	0.970
43	0.275	0.929	0.231	0.950	0.205	0.960	0.188	0.966	0.176	0.970
44	0.274	0.929	0.230	0.950	0.204	0.960				
$s_{r,n}$	0.272	0.929	0.229	0.950	0.203	0.960	0.187	0.966	0.175	0.970
true min availability	0.929		0.950		0.960		0.966		0.970	
true average cost	16.133		15.117		14.968		15.063		15.235	
efficiency	0.928		0.990		1.000		0.994		0.983	

Table 3. *The minimum average cost and the optimal IRR policy as p varies for $W(a,b_i)$.*

p	0.910		0.930		0.950*		0.970		0.990	
availability	0.960		0.960		0.960		0.960		0.960	
average cost	15.129		15.048		14.968*		14.891		14.816	
number of inspection	44		44		44		44		44	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	0.286	0.960	0.286	0.960	0.286	0.960	0.286	0.960	0.286	0.960
2	0.270	0.962	0.273	0.961	0.276	0.960	0.279	0.959	0.282	0.958
3	0.268	0.962	0.270	0.961	0.273	0.960	0.276	0.959	0.279	0.958
4	0.265	0.962	0.268	0.961	0.271	0.960	0.273	0.959	0.276	0.958
5	0.263	0.962	0.265	0.961	0.268	0.960	0.271	0.959	0.274	0.958
6	0.260	0.962	0.263	0.961	0.266	0.960	0.268	0.959	0.271	0.958
7	0.258	0.962	0.261	0.961	0.263	0.960	0.266	0.959	0.269	0.958
8	0.256	0.962	0.258	0.961	0.261	0.960	0.264	0.959	0.266	0.958
9	0.253	0.962	0.256	0.961	0.259	0.960	0.261	0.959	0.264	0.958
10	0.251	0.962	0.254	0.961	0.257	0.960	0.259	0.959	0.262	0.958
11	0.249	0.962	0.252	0.961	0.254	0.960	0.257	0.959	0.260	0.958
12	0.247	0.962	0.250	0.961	0.252	0.960	0.255	0.959	0.257	0.958
13	0.245	0.962	0.248	0.961	0.250	0.960	0.253	0.959	0.255	0.958
14	0.243	0.962	0.246	0.961	0.248	0.960	0.251	0.959	0.253	0.958
15	0.241	0.962	0.244	0.961	0.246	0.960	0.249	0.959	0.251	0.958
16	0.239	0.962	0.242	0.961	0.244	0.960	0.247	0.959	0.249	0.958
17	0.237	0.962	0.240	0.961	0.243	0.960	0.245	0.959	0.247	0.958
18	0.236	0.962	0.238	0.961	0.241	0.960	0.243	0.959	0.246	0.958
19	0.234	0.962	0.236	0.961	0.239	0.960	0.241	0.959	0.244	0.958
20	0.232	0.962	0.235	0.961	0.237	0.960	0.240	0.959	0.242	0.958
21	0.231	0.962	0.233	0.961	0.235	0.960	0.238	0.959	0.240	0.958
22	0.229	0.962	0.231	0.961	0.234	0.960	0.236	0.959	0.239	0.958
23	0.227	0.962	0.230	0.961	0.232	0.960	0.235	0.959	0.237	0.958
24	0.226	0.962	0.228	0.961	0.231	0.960	0.233	0.959	0.235	0.958
25	0.224	0.962	0.227	0.961	0.229	0.960	0.231	0.959	0.234	0.958
26	0.223	0.962	0.225	0.961	0.228	0.960	0.230	0.959	0.232	0.958
27	0.221	0.962	0.224	0.961	0.226	0.960	0.228	0.959	0.231	0.958
28	0.220	0.962	0.222	0.961	0.225	0.960	0.227	0.959	0.229	0.958
29	0.218	0.962	0.221	0.961	0.223	0.960	0.225	0.959	0.228	0.958
30	0.217	0.962	0.219	0.961	0.222	0.960	0.224	0.959	0.226	0.958
31	0.216	0.962	0.218	0.961	0.220	0.960	0.223	0.959	0.225	0.958
32	0.214	0.962	0.217	0.961	0.219	0.960	0.221	0.959	0.223	0.958
33	0.213	0.962	0.215	0.961	0.218	0.960	0.220	0.959	0.222	0.958
34	0.212	0.962	0.214	0.961	0.216	0.960	0.218	0.959	0.221	0.958
35	0.210	0.962	0.213	0.961	0.215	0.960	0.217	0.959	0.219	0.958
36	0.209	0.962	0.211	0.961	0.214	0.960	0.216	0.959	0.218	0.958
37	0.208	0.962	0.210	0.961	0.212	0.960	0.215	0.959	0.217	0.958
38	0.207	0.962	0.209	0.961	0.211	0.960	0.213	0.959	0.216	0.958
39	0.206	0.962	0.208	0.961	0.210	0.960	0.212	0.959	0.214	0.958
40	0.204	0.962	0.207	0.961	0.209	0.960	0.211	0.959	0.213	0.958
41	0.203	0.962	0.206	0.961	0.208	0.960	0.210	0.959	0.212	0.958
42	0.202	0.962	0.204	0.961	0.207	0.960	0.209	0.959	0.211	0.958
43	0.201	0.962	0.203	0.961	0.205	0.960	0.208	0.959	0.210	0.958
44	0.200	0.962	0.202	0.961	0.204	0.960	0.206	0.959	0.208	0.958
$s_{r,n}$	0.199	0.962	0.201	0.961	0.203	0.960	0.205	0.959	0.207	0.958
true min availability	0.960		0.960		0.960		0.959		0.958	
true average cost	15.131		15.048		14.968		14.970		14.974	
efficiency	0.989		0.995		1.000		1.000		1.000	

Table 4. The minimum average cost and the optimal IRR policy as p' varies for $W(a,b_i)$.

p'	0.910		0.930		0.950*		0.970		0.990	
availability	0.960		0.960		0.960		0.960		0.958	
average cost	15.228		15.100		14.968*		14.835		14.699	
number of inspection	44		44		44		43		43	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	0.286	0.960	0.286	0.960	0.286	0.960	0.286	0.960	0.293	0.958
2	0.276	0.960	0.276	0.960	0.276	0.960	0.276	0.960	0.283	0.958
3	0.273	0.960	0.273	0.960	0.273	0.960	0.273	0.960	0.280	0.958
4	0.271	0.960	0.271	0.960	0.271	0.960	0.271	0.960	0.277	0.958
5	0.268	0.960	0.268	0.960	0.268	0.960	0.268	0.960	0.275	0.958
6	0.266	0.960	0.266	0.960	0.266	0.960	0.266	0.960	0.272	0.958
7	0.263	0.960	0.263	0.960	0.263	0.960	0.263	0.960	0.270	0.958
8	0.261	0.960	0.261	0.960	0.261	0.960	0.261	0.960	0.268	0.958
9	0.259	0.960	0.259	0.960	0.259	0.960	0.259	0.960	0.265	0.958
10	0.257	0.960	0.257	0.960	0.257	0.960	0.257	0.960	0.263	0.958
11	0.254	0.960	0.254	0.960	0.254	0.960	0.254	0.960	0.261	0.958
12	0.252	0.960	0.252	0.960	0.252	0.960	0.252	0.960	0.259	0.958
13	0.250	0.960	0.250	0.960	0.250	0.960	0.250	0.960	0.257	0.958
14	0.248	0.960	0.248	0.960	0.248	0.960	0.248	0.960	0.255	0.958
15	0.246	0.960	0.246	0.960	0.246	0.960	0.246	0.960	0.253	0.958
16	0.244	0.960	0.244	0.960	0.244	0.960	0.244	0.960	0.251	0.958
17	0.243	0.960	0.243	0.960	0.243	0.960	0.243	0.960	0.249	0.958
18	0.241	0.960	0.241	0.960	0.241	0.960	0.241	0.960	0.247	0.958
19	0.239	0.960	0.239	0.960	0.239	0.960	0.239	0.960	0.245	0.958
20	0.237	0.960	0.237	0.960	0.237	0.960	0.237	0.960	0.243	0.958
21	0.235	0.960	0.235	0.960	0.235	0.960	0.235	0.960	0.241	0.958
22	0.234	0.960	0.234	0.960	0.234	0.960	0.234	0.960	0.240	0.958
23	0.232	0.960	0.232	0.960	0.232	0.960	0.232	0.960	0.238	0.958
24	0.231	0.960	0.231	0.960	0.231	0.960	0.231	0.960	0.236	0.958
25	0.229	0.960	0.229	0.960	0.229	0.960	0.229	0.960	0.235	0.958
26	0.228	0.960	0.228	0.960	0.228	0.960	0.228	0.960	0.233	0.958
27	0.226	0.960	0.226	0.960	0.226	0.960	0.226	0.960	0.232	0.958
28	0.225	0.960	0.225	0.960	0.225	0.960	0.225	0.960	0.230	0.958
29	0.223	0.960	0.223	0.960	0.223	0.960	0.223	0.960	0.229	0.958
30	0.222	0.960	0.222	0.960	0.222	0.960	0.222	0.960	0.227	0.958
31	0.220	0.960	0.220	0.960	0.220	0.960	0.220	0.960	0.226	0.958
32	0.219	0.960	0.219	0.960	0.219	0.960	0.219	0.960	0.224	0.958
33	0.218	0.960	0.218	0.960	0.218	0.960	0.218	0.960	0.223	0.958
34	0.216	0.960	0.216	0.960	0.216	0.960	0.216	0.960	0.222	0.958
35	0.215	0.960	0.215	0.960	0.215	0.960	0.215	0.960	0.220	0.958
36	0.214	0.960	0.214	0.960	0.214	0.960	0.214	0.960	0.219	0.958
37	0.212	0.960	0.212	0.960	0.212	0.960	0.212	0.960	0.218	0.958
38	0.211	0.960	0.211	0.960	0.211	0.960	0.211	0.960	0.217	0.958
39	0.210	0.960	0.210	0.960	0.210	0.960	0.210	0.960	0.215	0.958
40	0.209	0.960	0.209	0.960	0.209	0.960	0.209	0.960	0.214	0.958
41	0.208	0.960	0.208	0.960	0.208	0.960	0.208	0.960	0.213	0.958
42	0.207	0.960	0.207	0.960	0.207	0.960	0.207	0.960	0.212	0.958
43	0.205	0.960	0.205	0.960	0.205	0.960	0.205	0.960	0.211	0.958
44	0.204	0.960	0.204	0.960	0.204	0.960				
$s_{r,n}$	0.203	0.960	0.203	0.960	0.203	0.960	0.204	0.960	0.209	0.958
true min availability	0.960		0.960		0.960		0.960		0.958	
true average cost	14.968		14.968		14.968		14.969		14,971	
efficiency	1.000		1.000		1.000		1.000		1.000	

Table 5. The minimum average cost and the optimal IRR policy as λ_i varies for $W(a,b_i)$.

λ_i	$0.005+5\times10^{-3}$ i		$0.0075+7.5\times10^{-3}$ i		$0.01+10^{-3}$ i*		$0.0125+1.25\times10^{-2}$i		$0.015+1.5\times10^{-2}$ i	
availability	0.958		0.960		0.960		0.960		0.960	
average cost	12.662		13.939		14.968*		15.842		16.608	
number of inspection	60		50		44		39		36	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	0.271	0.964	0.278	0.962	0.286	0.960	0.293	0.958	0.300	0.956
2	0.261	0.964	0.269	0.962	0.276	0.960	0.283	0.958	0.290	0.956
3	0.259	0.964	0.266	0.962	0.273	0.960	0.280	0.958	0.287	0.956
4	0.256	0.964	0.264	0.962	0.271	0.960	0.277	0.958	0.284	0.956
5	0.254	0.964	0.261	0.962	0.268	0.960	0.275	0.958	0.282	0.956
6	0.252	0.964	0.259	0.962	0.266	0.960	0.272	0.958	0.279	0.956
7	0.250	0.964	0.256	0.962	0.263	0.960	0.270	0.958	0.276	0.956
8	0.247	0.964	0.254	0.962	0.261	0.960	0.268	0.958	0.274	0.956
9	0.245	0.964	0.252	0.962	0.259	0.960	0.265	0.958	0.272	0.956
10	0.243	0.964	0.250	0.962	0.257	0.960	0.263	0.958	0.269	0.956
11	0.241	0.964	0.248	0.962	0.254	0.960	0.261	0.958	0.267	0.956
12	0.239	0.964	0.246	0.962	0.252	0.960	0.259	0.958	0.265	0.956
13	0.237	0.964	0.244	0.962	0.250	0.960	0.257	0.958	0.263	0.956
14	0.235	0.964	0.242	0.962	0.248	0.960	0.255	0.958	0.261	0.956
15	0.233	0.964	0.240	0.962	0.246	0.960	0.253	0.958	0.259	0.956
16	0.232	0.964	0.238	0.962	0.244	0.960	0.251	0.958	0.257	0.956
17	0.230	0.964	0.236	0.962	0.243	0.960	0.249	0.958	0.255	0.956
18	0.228	0.964	0.234	0.962	0.241	0.960	0.247	0.958	0.253	0.956
19	0.226	0.964	0.233	0.962	0.239	0.960	0.245	0.958	0.251	0.956
20	0.225	0.964	0.231	0.962	0.237	0.960	0.243	0.958	0.249	0.956
21	0.223	0.964	0.229	0.962	0.235	0.960	0.241	0.958	0.247	0.956
22	0.222	0.964	0.228	0.962	0.234	0.960	0.240	0.958	0.246	0.956
23	0.220	0.964	0.226	0.962	0.232	0.960	0.238	0.958	0.244	0.956
24	0.219	0.964	0.225	0.962	0.231	0.960	0.236	0.958	0.242	0.956
25	0.217	0.964	0.223	0.962	0.229	0.960	0.235	0.958	0.240	0.956
26	0.216	0.964	0.222	0.962	0.228	0.960	0.233	0.958	0.239	0.956
27	0.214	0.964	0.220	0.962	0.226	0.960	0.232	0.958	0.237	0.956
28	0.213	0.964	0.219	0.962	0.225	0.960	0.230	0.958	0.236	0.956
29	0.211	0.964	0.217	0.962	0.223	0.960	0.229	0.958	0.234	0.956
30	0.210	0.964	0.216	0.962	0.222	0.960	0.227	0.958	0.233	0.956
31	0.209	0.964	0.215	0.962	0.220	0.960	0.226	0.958	0.231	0.956
32	0.207	0.964	0.213	0.962	0.219	0.960	0.224	0.958	0.230	0.956
33	0.206	0.964	0.212	0.962	0.218	0.960	0.223	0.958	0.228	0.956
34	0.205	0.964	0.211	0.962	0.216	0.960	0.222	0.958	0.227	0.956
35	0.204	0.964	0.209	0.962	0.215	0.960	0.220	0.958	0.226	0.956
36	0.203	0.964	0.208	0.962	0.214	0.960	0.219	0.958	0.224	0.956
37	0.201	0.964	0.207	0.962	0.212	0.960	0.218	0.958		
38	0.200	0.964	0.206	0.962	0.211	0.960	0.217	0.958		
39	0.199	0.964	0.205	0.962	0.210	0.960	0.215	0.958		
40	0.198	0.964	0.203	0.962	0.209	0.960				
41	0.197	0.964	0.202	0.962	0.208	0.960				
42	0.196	0.964	0.201	0.962	0.207	0.960				
43	0.195	0.964	0.200	0.962	0.205	0.960				
44	0.194	0.964	0.199	0.962	0.204	0.960				
45	0.193	0.964	0.198	0.962						
46	0.192	0.964	0.197	0.962						
47	0.191	0.964	0.196	0.962						
48	0.190	0.964	0.195	0.962						
49	0.189	0.964	0.194	0.962						
50	0.188	0.964	0.193	0.962						
51	0.187	0.964								

Table 5. *(Continue)*

52	0.186	0.964								
53	0.185	0.964								
54	0.184	0.964								
55	0.183	0.964								
56	0.182	0.964								
57	0.181	0.964								
58	0.181	0.964								
59	0.180	0.964								
60	0.179	0.964								
$s_{r,n}$	0.178	0.964	0.192	0.962	0.203	0.960	0.214	0.958	0.223	0.956
true min availability	0.964		0.962		0.960		0.958		0.956	
true average cost	15.386		15.047		14.968		15.014		15.110	
efficiency	0.973		0.995		1.000		0.997		0.991	

Table 6. The minimum average cost and the optimal IRR policy as μ_i varies for $W(a,b_i)$.

μ_i	$0.05+5\times10^{-3}\,i$		$0.075+7.5\times10^{-3}\,i$		$0.1+10^{-3}\,i$*		$0.125+1.25\times10^{-2}\,i$		$0.15+1.5\times10^{-2}\,i$	
availability	0.958		0.960		0.960		0.960		0.960	
average cost	14.665		14.819		14.968*		15.115		15.259	
number of inspection	44		44		44		43		43	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	0.293	0.958	0.286	0.960	0.286	0.960	0.286	0.960	0.286	0.960
2	0.283	0.958	0.276	0.960	0.276	0.960	0.276	0.960	0.276	0.960
3	0.280	0.958	0.273	0.960	0.273	0.960	0.273	0.960	0.273	0.960
4	0.277	0.958	0.271	0.960	0.271	0.960	0.271	0.960	0.271	0.960
5	0.275	0.958	0.268	0.960	0.268	0.960	0.268	0.960	0.268	0.960
6	0.272	0.958	0.266	0.960	0.266	0.960	0.266	0.960	0.266	0.960
7	0.270	0.958	0.263	0.960	0.263	0.960	0.263	0.960	0.263	0.960
8	0.268	0.958	0.261	0.960	0.261	0.960	0.261	0.960	0.261	0.960
9	0.265	0.958	0.259	0.960	0.259	0.960	0.259	0.960	0.259	0.960
10	0.263	0.958	0.257	0.960	0.257	0.960	0.257	0.960	0.257	0.960
11	0.261	0.958	0.254	0.960	0.254	0.960	0.254	0.960	0.254	0.960
12	0.259	0.958	0.252	0.960	0.252	0.960	0.252	0.960	0.252	0.960
13	0.257	0.958	0.250	0.960	0.250	0.960	0.250	0.960	0.250	0.960
14	0.255	0.958	0.248	0.960	0.248	0.960	0.248	0.960	0.248	0.960
15	0.253	0.958	0.246	0.960	0.246	0.960	0.246	0.960	0.246	0.960
16	0.251	0.958	0.244	0.960	0.244	0.960	0.244	0.960	0.244	0.960
17	0.249	0.958	0.243	0.960	0.243	0.960	0.243	0.960	0.243	0.960
18	0.247	0.958	0.241	0.960	0.241	0.960	0.241	0.960	0.241	0.960
19	0.245	0.958	0.239	0.960	0.239	0.960	0.239	0.960	0.239	0.960
20	0.243	0.958	0.237	0.960	0.237	0.960	0.237	0.960	0.237	0.960
21	0.241	0.958	0.235	0.960	0.235	0.960	0.235	0.960	0.235	0.960
22	0.240	0.958	0.234	0.960	0.234	0.960	0.234	0.960	0.234	0.960
23	0.238	0.958	0.232	0.960	0.232	0.960	0.232	0.960	0.232	0.960
24	0.236	0.958	0.231	0.960	0.231	0.960	0.231	0.960	0.231	0.960
25	0.235	0.958	0.229	0.960	0.229	0.960	0.229	0.960	0.229	0.960
26	0.233	0.958	0.228	0.960	0.228	0.960	0.228	0.960	0.228	0.960
27	0.232	0.958	0.226	0.960	0.226	0.960	0.226	0.960	0.226	0.960
28	0.230	0.958	0.225	0.960	0.225	0.960	0.225	0.960	0.225	0.960
29	0.229	0.958	0.223	0.960	0.223	0.960	0.223	0.960	0.223	0.960
30	0.227	0.958	0.222	0.960	0.222	0.960	0.222	0.960	0.222	0.960
31	0.226	0.958	0.220	0.960	0.220	0.960	0.220	0.960	0.220	0.960
32	0.224	0.958	0.219	0.960	0.219	0.960	0.219	0.960	0.219	0.960
33	0.223	0.958	0.218	0.960	0.218	0.960	0.218	0.960	0.218	0.960
34	0.222	0.958	0.216	0.960	0.216	0.960	0.216	0.960	0.216	0.960
35	0.220	0.958	0.215	0.960	0.215	0.960	0.215	0.960	0.215	0.960
36	0.219	0.958	0.214	0.960	0.214	0.960	0.214	0.960	0.214	0.960
37	0.218	0.958	0.212	0.960	0.212	0.960	0.212	0.960	0.212	0.960
38	0.217	0.958	0.211	0.960	0.211	0.960	0.211	0.960	0.211	0.960
39	0.215	0.958	0.210	0.960	0.210	0.960	0.210	0.960	0.210	0.960
40	0.214	0.958	0.209	0.960	0.209	0.960	0.209	0.960	0.209	0.960
41	0.213	0.958	0.208	0.960	0.208	0.960	0.208	0.960	0.208	0.960
42	0.212	0.958	0.207	0.960	0.207	0.960	0.207	0.960	0.207	0.960
43	0.211	0.958	0.205	0.960	0.205	0.960	0.205	0.960	0.205	0.960
44	0.209	0.958	0.204	0.960	0.204	0.960				
$s_{r,n}$	0.208	0.958	0.203	0.960	0.203	0.960	0.204	0.960	0.204	0.960
true min availability	0.958		0.960		0.960		0.960		0.960	
true average cost	14.971		14.968		14.968		14.969		14.969	
efficiency	1.000		1.000		1.000		1.000		1.000	

Table 7. *The minimum average cost and the optimal IRR policy as α varies for $\Gamma(\alpha,\beta)$.*

α	3		4		5*		6		7	
availability	0.950		0.970		0.980		0.986		0.990	
average cost	10.577		7.443		5.597*		4.409		3.595	
number of inspection	40		41		41		41		41	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	1.635	0.998	2.310	0.993	3.059	0.980	3.850	0.954	4.660	0.913
2	1.554	0.999	2.214	0.994	2.947	0.980	3.723	0.953	4.520	0.910
3	1.538	0.999	2.192	0.994	2.918	0.980	3.687	0.953	4.476	0.910
4	1.523	0.999	2.171	0.994	2.890	0.980	3.651	0.953	4.432	0.910
5	1.509	0.999	2.150	0.994	2.862	0.980	3.616	0.953	4.390	0.910
6	1.494	0.999	2.129	0.994	2.835	0.980	3.581	0.953	4.348	0.910
7	1.480	0.999	2.109	0.994	2.808	0.980	3.548	0.953	4.307	0.910
8	1.466	0.999	2.089	0.994	2.782	0.980	3.514	0.953	4.267	0.910
9	1.453	0.999	2.070	0.994	2.756	0.980	3.482	0.953	4.227	0.910
10	1.440	0.999	2.051	0.994	2.731	0.980	3.450	0.953	4.188	0.910
11	1.426	0.999	2.032	0.994	2.706	0.980	3.419	0.953	4.150	0.910
12	1.414	0.999	2.014	0.994	2.682	0.980	3.388	0.953	4.113	0.910
13	1.401	0.999	1.996	0.994	2.658	0.980	3.357	0.953	4.076	0.910
14	1.389	0.999	1.979	0.994	2.634	0.980	3.328	0.953	4.040	0.910
15	1.376	0.999	1.961	0.994	2.611	0.980	3.299	0.953	4.005	0.910
16	1.364	0.999	1.944	0.994	2.589	0.980	3.270	0.953	3.970	0.910
17	1.353	0.999	1.927	0.994	2.566	0.980	3.242	0.953	3.936	0.910
18	1.341	0.999	1.911	0.994	2.544	0.980	3.214	0.953	3.902	0.910
19	1.330	0.999	1.895	0.994	2.523	0.980	3.187	0.953	3.869	0.910
20	1.319	0.999	1.879	0.994	2.502	0.980	3.160	0.953	3.836	0.910
21	1.308	0.999	1.863	0.994	2.481	0.980	3.134	0.953	3.804	0.910
22	1.297	0.999	1.848	0.994	2.460	0.980	3.108	0.953	3.773	0.910
23	1.286	0.999	1.833	0.994	2.440	0.980	3.082	0.953	3.742	0.910
24	1.276	0.999	1.818	0.994	2.420	0.980	3.057	0.953	3.712	0.910
25	1.265	0.999	1.803	0.994	2.401	0.980	3.033	0.953	3.682	0.910
26	1.255	0.999	1.789	0.994	2.381	0.980	3.008	0.953	3.652	0.910
27	1.245	0.999	1.774	0.994	2.363	0.980	2.984	0.953	3.623	0.910
28	1.236	0.999	1.760	0.994	2.344	0.980	2.961	0.953	3.595	0.910
29	1.226	0.999	1.747	0.994	2.326	0.980	2.938	0.953	3.567	0.910
30	1.216	0.999	1.733	0.994	2.308	0.980	2.915	0.953	3.539	0.910
31	1.207	0.999	1.720	0.994	2.290	0.980	2.893	0.953	3.512	0.910
32	1.198	0.999	1.707	0.994	2.272	0.980	2.871	0.953	3.485	0.910
33	1.189	0.999	1.694	0.994	2.255	0.980	2.849	0.953	3.459	0.910
34	1.180	0.999	1.681	0.994	2.238	0.980	2.827	0.953	3.433	0.910
35	1.171	0.999	1.668	0.994	2.222	0.980	2.806	0.953	3.407	0.910
36	1.162	0.999	1.656	0.994	2.205	0.980	2.785	0.953	3.382	0.910
37	1.154	0.999	1.644	0.994	2.189	0.980	2.765	0.953	3.357	0.910
38	1.145	0.999	1.632	0.994	2.173	0.980	2.745	0.953	3.332	0.910
39	1.137	0.999	1.620	0.994	2.157	0.980	2.725	0.953	3.308	0.910
40	1.129	0.999	1.608	0.994	2.142	0.980	2.705	0.953	3.284	0.910
41			1.597	0.994	2.126	0.980	2.686	0.953	3.261	0.910
$s_{r,n}$	1.121	0.999	1.586	0.994	2.111	0.980	2.667	0.953	3.238	0.910
true min availability	0.998		0.993		0.980		0.953		0.910	
true average cost	7.951		6.195		5.597		6.133		7.749	
efficiency	0.704		0.903		1.000		0.912		0.722	

Table 8. *The minimum average cost and the optimal IRR policy as* β *varies for* $\Gamma(\alpha,\beta)$.

β	0.3		0.4		0.5*		0.6		0.7	
availability	0.986		0.982		0.980		0.976		0.974	
average cost	3.829		4.749		5.597*		6.388		7.131	
number of inspection	41		41		41		41		41	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	4.645	0.914	3.719	0.959	3.059	0.980	2.676	0.988	2.344	0.993
2	4.478	0.913	3.584	0.959	2.947	0.980	2.578	0.988	2.257	0.993
3	4.434	0.913	3.549	0.959	2.918	0.980	2.553	0.988	2.235	0.993
4	4.391	0.913	3.514	0.959	2.890	0.980	2.528	0.988	2.214	0.993
5	4.349	0.913	3.481	0.959	2.862	0.980	2.503	0.988	2.192	0.993
6	4.307	0.913	3.447	0.959	2.835	0.980	2.480	0.988	2.171	0.993
7	4.267	0.913	3.415	0.959	2.808	0.980	2.456	0.988	2.151	0.993
8	4.227	0.913	3.383	0.959	2.782	0.980	2.433	0.988	2.131	0.993
9	4.188	0.913	3.352	0.959	2.756	0.980	2.411	0.988	2.111	0.993
10	4.149	0.913	3.321	0.959	2.731	0.980	2.389	0.988	2.092	0.993
11	4.112	0.913	3.291	0.959	2.706	0.980	2.367	0.988	2.073	0.993
12	4.075	0.913	3.261	0.959	2.682	0.980	2.346	0.988	2.054	0.993
13	4.038	0.913	3.232	0.959	2.658	0.980	2.325	0.988	2.036	0.993
14	4.002	0.913	3.203	0.959	2.634	0.980	2.304	0.988	2.018	0.993
15	3.967	0.913	3.175	0.959	2.611	0.980	2.284	0.988	2.000	0.993
16	3.933	0.913	3.148	0.959	2.589	0.980	2.264	0.988	1.983	0.993
17	3.899	0.913	3.120	0.959	2.566	0.980	2.244	0.988	1.966	0.993
18	3.866	0.913	3.094	0.959	2.544	0.980	2.225	0.988	1.949	0.993
19	3.833	0.913	3.068	0.959	2.523	0.980	2.206	0.988	1.932	0.993
20	3.801	0.913	3.042	0.959	2.502	0.980	2.188	0.988	1.916	0.993
21	3.769	0.913	3.016	0.959	2.481	0.980	2.170	0.988	1.900	0.993
22	3.738	0.913	2.992	0.959	2.460	0.980	2.152	0.988	1.884	0.993
23	3.707	0.913	2.967	0.959	2.440	0.980	2.134	0.988	1.869	0.993
24	3.677	0.913	2.943	0.959	2.420	0.980	2.117	0.988	1.854	0.993
25	3.647	0.913	2.919	0.959	2.401	0.980	2.100	0.988	1.839	0.993
26	3.618	0.913	2.896	0.959	2.381	0.980	2.083	0.988	1.824	0.993
27	3.590	0.913	2.873	0.959	2.363	0.980	2.066	0.988	1.810	0.993
28	3.561	0.913	2.850	0.959	2.344	0.980	2.050	0.988	1.795	0.993
29	3.533	0.913	2.828	0.959	2.326	0.980	2.034	0.988	1.781	0.993
30	3.506	0.913	2.806	0.959	2.308	0.980	2.018	0.988	1.767	0.993
31	3.479	0.913	2.784	0.959	2.290	0.980	2.003	0.988	1.754	0.993
32	3.453	0.913	2.763	0.959	2.272	0.980	1.987	0.988	1.740	0.993
33	3.426	0.913	2.742	0.959	2.255	0.980	1.972	0.988	1.727	0.993
34	3.401	0.913	2.722	0.959	2.238	0.980	1.958	0.988	1.714	0.993
35	3.375	0.913	2.701	0.959	2.222	0.980	1.943	0.988	1.702	0.993
36	3.350	0.913	2.681	0.959	2.205	0.980	1.929	0.988	1.689	0.993
37	3.326	0.913	2.662	0.959	2.189	0.980	1.914	0.988	1.676	0.993
38	3.301	0.913	2.642	0.959	2.173	0.980	1.900	0.988	1.664	0.993
39	3.277	0.913	2.623	0.959	2.157	0.980	1.887	0.988	1.652	0.993
40	3.254	0.913	2.604	0.959	2.142	0.980	1.873	0.988	1.640	0.993
41	3.231	0.913	2.586	0.959	2.126	0.980	1.860	0.988	1.629	0.993
$s_{r,n}$	3.208	0.913	2.567	0.959	2.111	0.980	1.847	0.988	1.617	0.993
true min availability	0.913		0.959		0.980		0.988		0.993	
true average cost	7.639		5.957		5.597		5.749		6.119	
efficiency	0.733		0.939		1.000		0.974		0.915	

Table 9. *The minimum average cost and the optimal IRR policy as p varies for* $\Gamma(\alpha,\beta)$.

p	0.810		0.855		0.9*		0.945		0.990	
availability	0.980		0.980		0.980		0.980		0.980	
average cost	5.699		5.646		5.597*		5.550		5.505	
number of inspection	41		41		41		41		41	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	3.059	0.980	3.059	0.980	3.059	0.980	3.059	0.980	3.059	0.980
2	2.869	0.982	2.909	0.981	2.947	0.980	2.985	0.979	3.021	0.978
3	2.841	0.982	2.880	0.981	2.918	0.980	2.955	0.979	2.991	0.978
4	2.813	0.982	2.852	0.981	2.890	0.980	2.927	0.979	2.962	0.978
5	2.786	0.982	2.825	0.981	2.862	0.980	2.899	0.979	2.934	0.978
6	2.759	0.982	2.798	0.981	2.835	0.980	2.871	0.979	2.906	0.978
7	2.733	0.982	2.772	0.981	2.808	0.980	2.844	0.979	2.878	0.978
8	2.708	0.982	2.746	0.981	2.782	0.980	2.817	0.979	2.851	0.978
9	2.683	0.982	2.720	0.981	2.756	0.980	2.791	0.979	2.825	0.978
10	2.658	0.982	2.695	0.981	2.731	0.980	2.766	0.979	2.799	0.978
11	2.634	0.982	2.671	0.981	2.706	0.980	2.741	0.979	2.774	0.978
12	2.610	0.982	2.647	0.981	2.682	0.980	2.716	0.979	2.749	0.978
13	2.587	0.982	2.623	0.981	2.658	0.980	2.692	0.979	2.724	0.978
14	2.564	0.982	2.600	0.981	2.634	0.980	2.668	0.979	2.700	0.978
15	2.542	0.982	2.577	0.981	2.611	0.980	2.644	0.979	2.676	0.978
16	2.519	0.982	2.555	0.981	2.589	0.980	2.621	0.979	2.653	0.978
17	2.498	0.982	2.533	0.981	2.566	0.980	2.599	0.979	2.630	0.978
18	2.476	0.982	2.511	0.981	2.544	0.980	2.577	0.979	2.608	0.978
19	2.455	0.982	2.490	0.981	2.523	0.980	2.555	0.979	2.586	0.978
20	2.435	0.982	2.469	0.981	2.502	0.980	2.533	0.979	2.564	0.978
21	2.414	0.982	2.448	0.981	2.481	0.980	2.512	0.979	2.543	0.978
22	2.394	0.982	2.428	0.981	2.460	0.980	2.491	0.979	2.522	0.978
23	2.375	0.982	2.408	0.981	2.440	0.980	2.471	0.979	2.501	0.978
24	2.356	0.982	2.388	0.981	2.420	0.980	2.451	0.979	2.481	0.978
25	2.337	0.982	2.369	0.981	2.401	0.980	2.431	0.979	2.461	0.978
26	2.318	0.982	2.350	0.981	2.381	0.980	2.412	0.979	2.441	0.978
27	2.299	0.982	2.332	0.981	2.363	0.980	2.393	0.979	2.422	0.978
28	2.281	0.982	2.313	0.981	2.344	0.980	2.374	0.979	2.402	0.978
29	2.264	0.982	2.295	0.981	2.326	0.980	2.355	0.979	2.384	0.978
30	2.246	0.982	2.277	0.981	2.308	0.980	2.337	0.979	2.365	0.978
31	2.229	0.982	2.260	0.981	2.290	0.980	2.319	0.979	2.347	0.978
32	2.212	0.982	2.243	0.981	2.272	0.980	2.301	0.979	2.329	0.978
33	2.195	0.982	2.226	0.981	2.255	0.980	2.284	0.979	2.311	0.978
34	2.178	0.982	2.209	0.981	2.238	0.980	2.267	0.979	2.294	0.978
35	2.162	0.982	2.192	0.981	2.222	0.980	2.250	0.979	2.277	0.978
36	2.146	0.982	2.176	0.981	2.205	0.980	2.233	0.979	2.260	0.978
37	2.130	0.982	2.160	0.981	2.189	0.980	2.217	0.979	2.243	0.978
38	2.115	0.982	2.144	0.981	2.173	0.980	2.200	0.979	2.227	0.978
39	2.100	0.982	2.129	0.981	2.157	0.980	2.184	0.979	2.211	0.978
40	2.084	0.982	2.114	0.981	2.142	0.980	2.169	0.979	2.195	0.978
41	2.070	0.982	2.098	0.981	2.126	0.980	2.153	0.979	2.179	0.978
$s_{r,n}$	2.055	0.982	2.084	0.981	2.111	0.980	2.138	0.979	2.164	0.978
true min availability	0.980		0.980		0.980		0.979		0.978	
true average cost	5.700		5.647		5.597		5.598		5.602	
efficiency	0.982		0.991		1.000		1.000		0.999	

Table 10. *The minimum average cost and the optimal IRR policy as p' varies for* $\Gamma(\alpha,\beta)$.

p'	0.810		0.855		0.9*		0.945		0.990	
availability	0.978		0.978		0.980		0.980		0.980	
average cost	5.889		5.746		5.597*		5.444		5.286	
number of inspection	39		40		41		42		44	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	3.138	0.978	3.138	0.978	3.059	0.980	3.059	0.980	3.059	0.980
2	3.022	0.978	3.022	0.978	2.947	0.980	2.947	0.980	2.947	0.980
3	2.993	0.978	2.993	0.978	2.918	0.980	2.918	0.980	2.918	0.980
4	2.964	0.978	2.964	0.978	2.890	0.980	2.890	0.980	2.890	0.980
5	2.935	0.978	2.935	0.978	2.862	0.980	2.862	0.980	2.862	0.980
6	2.907	0.978	2.907	0.978	2.835	0.980	2.835	0.980	2.835	0.980
7	2.880	0.978	2.880	0.978	2.808	0.980	2.808	0.980	2.808	0.980
8	2.853	0.978	2.853	0.978	2.782	0.980	2.782	0.980	2.782	0.980
9	2.827	0.978	2.827	0.978	2.756	0.980	2.756	0.980	2.756	0.980
10	2.801	0.978	2.801	0.978	2.731	0.980	2.731	0.980	2.731	0.980
11	2.775	0.978	2.775	0.978	2.706	0.980	2.706	0.980	2.706	0.980
12	2.750	0.978	2.750	0.978	2.682	0.980	2.682	0.980	2.682	0.980
13	2.726	0.978	2.726	0.978	2.658	0.980	2.658	0.980	2.658	0.980
14	2.702	0.978	2.702	0.978	2.634	0.980	2.634	0.980	2.634	0.980
15	2.678	0.978	2.678	0.978	2.611	0.980	2.611	0.980	2.611	0.980
16	2.655	0.978	2.655	0.978	2.589	0.980	2.589	0.980	2.589	0.980
17	2.632	0.978	2.632	0.978	2.566	0.980	2.566	0.980	2.566	0.980
18	2.609	0.978	2.609	0.978	2.544	0.980	2.544	0.980	2.544	0.980
19	2.587	0.978	2.587	0.978	2.523	0.980	2.523	0.980	2.523	0.980
20	2.565	0.978	2.565	0.978	2.502	0.980	2.502	0.980	2.502	0.980
21	2.544	0.978	2.544	0.978	2.481	0.980	2.481	0.980	2.481	0.980
22	2.523	0.978	2.523	0.978	2.460	0.980	2.460	0.980	2.460	0.980
23	2.502	0.978	2.502	0.978	2.440	0.980	2.440	0.980	2.440	0.980
24	2.482	0.978	2.482	0.978	2.420	0.980	2.420	0.980	2.420	0.980
25	2.462	0.978	2.462	0.978	2.401	0.980	2.401	0.980	2.401	0.980
26	2.442	0.978	2.442	0.978	2.381	0.980	2.381	0.980	2.381	0.980
27	2.423	0.978	2.423	0.978	2.363	0.980	2.363	0.980	2.363	0.980
28	2.404	0.978	2.404	0.978	2.344	0.980	2.344	0.980	2.344	0.980
29	2.385	0.978	2.385	0.978	2.326	0.980	2.326	0.980	2.326	0.980
30	2.366	0.978	2.366	0.978	2.308	0.980	2.308	0.980	2.308	0.980
31	2.348	0.978	2.348	0.978	2.290	0.980	2.290	0.980	2.290	0.980
32	2.330	0.978	2.330	0.978	2.272	0.980	2.272	0.980	2.272	0.980
33	2.313	0.978	2.313	0.978	2.255	0.980	2.255	0.980	2.255	0.980
34	2.295	0.978	2.295	0.978	2.238	0.980	2.238	0.980	2.238	0.980
35	2.278	0.978	2.278	0.978	2.222	0.980	2.222	0.980	2.222	0.980
36	2.261	0.978	2.261	0.978	2.205	0.980	2.205	0.980	2.205	0.980
37	2.245	0.978	2.245	0.978	2.189	0.980	2.189	0.980	2.189	0.980
38	2.228	0.978	2.228	0.978	2.173	0.980	2.173	0.980	2.173	0.980
39	2.212	0.978	2.212	0.978	2.157	0.980	2.157	0.980	2.157	0.980
40			2.196	0.978	2.142	0.980	2.142	0.980	2.142	0.980
41					2.126	0.980	2.126	0.980	2.126	0.980
42							2.111	0.980	2.111	0.980
43									2.096	0.980
44									2.082	0.980
$s_{r,n}$	2.196	0.978	2.181	0.978	2.111	0.980	2.096	0.980	2.067	0.980
true min availability	0.978		0.978		0.980		0.980		0.980	
true average cost	5.603		5.601		5.597		5.597		5.603	
efficiency	0.999		0.999		1.000		1.000		0.999	

Table 11. *The minimum average cost and the optimal IRR policy as λ_i varies for $\Gamma(\alpha,\beta)$.*

λ_i	0.10		0.15		0.2*		0.25		0.30	
availability	0.978		0.978		0.980		0.980		0.980	
average cost	5.759		5.678		5.597*		5.518		5.442	
number of inspection	41		41		41		41		42	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	3.138	0.978	3.138	0.978	3.059	0.980	3.059	0.980	3.059	0.980
2	3.022	0.978	3.022	0.978	2.947	0.980	2.947	0.980	2.947	0.980
3	2.993	0.978	2.993	0.978	2.918	0.980	2.918	0.980	2.918	0.980
4	2.964	0.978	2.964	0.978	2.890	0.980	2.890	0.980	2.890	0.980
5	2.935	0.978	2.935	0.978	2.862	0.980	2.862	0.980	2.862	0.980
6	2.907	0.978	2.907	0.978	2.835	0.980	2.835	0.980	2.835	0.980
7	2.880	0.978	2.880	0.978	2.808	0.980	2.808	0.980	2.808	0.980
8	2.853	0.978	2.853	0.978	2.782	0.980	2.782	0.980	2.782	0.980
9	2.827	0.978	2.827	0.978	2.756	0.980	2.756	0.980	2.756	0.980
10	2.801	0.978	2.801	0.978	2.731	0.980	2.731	0.980	2.731	0.980
11	2.775	0.978	2.775	0.978	2.706	0.980	2.706	0.980	2.706	0.980
12	2.750	0.978	2.750	0.978	2.682	0.980	2.682	0.980	2.682	0.980
13	2.726	0.978	2.726	0.978	2.658	0.980	2.658	0.980	2.658	0.980
14	2.702	0.978	2.702	0.978	2.634	0.980	2.634	0.980	2.634	0.980
15	2.678	0.978	2.678	0.978	2.611	0.980	2.611	0.980	2.611	0.980
16	2.655	0.978	2.655	0.978	2.589	0.980	2.589	0.980	2.589	0.980
17	2.632	0.978	2.632	0.978	2.566	0.980	2.566	0.980	2.566	0.980
18	2.609	0.978	2.609	0.978	2.544	0.980	2.544	0.980	2.544	0.980
19	2.587	0.978	2.587	0.978	2.523	0.980	2.523	0.980	2.523	0.980
20	2.565	0.978	2.565	0.978	2.502	0.980	2.502	0.980	2.502	0.980
21	2.544	0.978	2.544	0.978	2.481	0.980	2.481	0.980	2.481	0.980
22	2.523	0.978	2.523	0.978	2.460	0.980	2.460	0.980	2.460	0.980
23	2.502	0.978	2.502	0.978	2.440	0.980	2.440	0.980	2.440	0.980
24	2.482	0.978	2.482	0.978	2.420	0.980	2.420	0.980	2.420	0.980
25	2.462	0.978	2.462	0.978	2.401	0.980	2.401	0.980	2.401	0.980
26	2.442	0.978	2.442	0.978	2.381	0.980	2.381	0.980	2.381	0.980
27	2.423	0.978	2.423	0.978	2.363	0.980	2.363	0.980	2.363	0.980
28	2.404	0.978	2.404	0.978	2.344	0.980	2.344	0.980	2.344	0.980
29	2.385	0.978	2.385	0.978	2.326	0.980	2.326	0.980	2.326	0.980
30	2.366	0.978	2.366	0.978	2.308	0.980	2.308	0.980	2.308	0.980
31	2.348	0.978	2.348	0.978	2.290	0.980	2.290	0.980	2.290	0.980
32	2.330	0.978	2.330	0.978	2.272	0.980	2.272	0.980	2.272	0.980
33	2.313	0.978	2.313	0.978	2.255	0.980	2.255	0.980	2.255	0.980
34	2.295	0.978	2.295	0.978	2.238	0.980	2.238	0.980	2.238	0.980
35	2.278	0.978	2.278	0.978	2.222	0.980	2.222	0.980	2.222	0.980
36	2.261	0.978	2.261	0.978	2.205	0.980	2.205	0.980	2.205	0.980
37	2.245	0.978	2.245	0.978	2.189	0.980	2.189	0.980	2.189	0.980
38	2.228	0.978	2.228	0.978	2.173	0.980	2.173	0.980	2.173	0.980
39	2.212	0.978	2.212	0.978	2.157	0.980	2.157	0.980	2.157	0.980
40	2.196	0.978	2.196	0.978	2.142	0.980	2.142	0.980	2.142	0.980
41	2.181	0.978	2.181	0.978	2.126	0.980	2.126	0.980	2.126	0.980
42									2.111	0.980
$s_{r,n}$	2.165	0.978	2.165	0.978	2.111	0.980	2.111	0.980	2.096	0.980
true min availability	0.978		0.978		0.980		0.980		0.980	
true average cost	5.600		5.600		5.597		5.597		5.597	
efficiency	0.999		0.999		1.000		1.000		1.000	

Table 12. *The minimum average cost and the optimal IRR policy as μ_i varies for $\Gamma(\alpha,\beta)$.*

μ_i	0.150		0.225		0.3*		0.375		0.450	
availability	0.980		0.980		0.980		0.980		0.980	
average cost	5.625		5.611		5.597*		5.583		5.569	
number of inspection	41		41		41		41		41	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	3.059	0.980	3.059	0.980	3.059	0.980	3.059	0.980	3.059	0.980
2	2.947	0.980	2.947	0.980	2.947	0.980	2.947	0.980	2.947	0.980
3	2.918	0.980	2.918	0.980	2.918	0.980	2.918	0.980	2.918	0.980
4	2.890	0.980	2.890	0.980	2.890	0.980	2.890	0.980	2.890	0.980
5	2.862	0.980	2.862	0.980	2.862	0.980	2.862	0.980	2.862	0.980
6	2.835	0.980	2.835	0.980	2.835	0.980	2.835	0.980	2.835	0.980
7	2.808	0.980	2.808	0.980	2.808	0.980	2.808	0.980	2.808	0.980
8	2.782	0.980	2.782	0.980	2.782	0.980	2.782	0.980	2.782	0.980
9	2.756	0.980	2.756	0.980	2.756	0.980	2.756	0.980	2.756	0.980
10	2.731	0.980	2.731	0.980	2.731	0.980	2.731	0.980	2.731	0.980
11	2.706	0.980	2.706	0.980	2.706	0.980	2.706	0.980	2.706	0.980
12	2.682	0.980	2.682	0.980	2.682	0.980	2.682	0.980	2.682	0.980
13	2.658	0.980	2.658	0.980	2.658	0.980	2.658	0.980	2.658	0.980
14	2.634	0.980	2.634	0.980	2.634	0.980	2.634	0.980	2.634	0.980
15	2.611	0.980	2.611	0.980	2.611	0.980	2.611	0.980	2.611	0.980
16	2.589	0.980	2.589	0.980	2.589	0.980	2.589	0.980	2.589	0.980
17	2.566	0.980	2.566	0.980	2.566	0.980	2.566	0.980	2.566	0.980
18	2.544	0.980	2.544	0.980	2.544	0.980	2.544	0.980	2.544	0.980
19	2.523	0.980	2.523	0.980	2.523	0.980	2.523	0.980	2.523	0.980
20	2.502	0.980	2.502	0.980	2.502	0.980	2.502	0.980	2.502	0.980
21	2.481	0.980	2.481	0.980	2.481	0.980	2.481	0.980	2.481	0.980
22	2.460	0.980	2.460	0.980	2.460	0.980	2.460	0.980	2.460	0.980
23	2.440	0.980	2.440	0.980	2.440	0.980	2.440	0.980	2.440	0.980
24	2.420	0.980	2.420	0.980	2.420	0.980	2.420	0.980	2.420	0.980
25	2.401	0.980	2.401	0.980	2.401	0.980	2.401	0.980	2.401	0.980
26	2.381	0.980	2.381	0.980	2.381	0.980	2.381	0.980	2.381	0.980
27	2.363	0.980	2.363	0.980	2.363	0.980	2.363	0.980	2.363	0.980
28	2.344	0.980	2.344	0.980	2.344	0.980	2.344	0.980	2.344	0.980
29	2.326	0.980	2.326	0.980	2.326	0.980	2.326	0.980	2.326	0.980
30	2.308	0.980	2.308	0.980	2.308	0.980	2.308	0.980	2.308	0.980
31	2.290	0.980	2.290	0.980	2.290	0.980	2.290	0.980	2.290	0.980
32	2.272	0.980	2.272	0.980	2.272	0.980	2.272	0.980	2.272	0.980
33	2.255	0.980	2.255	0.980	2.255	0.980	2.255	0.980	2.255	0.980
34	2.238	0.980	2.238	0.980	2.238	0.980	2.238	0.980	2.238	0.980
35	2.222	0.980	2.222	0.980	2.222	0.980	2.222	0.980	2.222	0.980
36	2.205	0.980	2.205	0.980	2.205	0.980	2.205	0.980	2.205	0.980
37	2.189	0.980	2.189	0.980	2.189	0.980	2.189	0.980	2.189	0.980
38	2.173	0.980	2.173	0.980	2.173	0.980	2.173	0.980	2.173	0.980
39	2.157	0.980	2.157	0.980	2.157	0.980	2.157	0.980	2.157	0.980
40	2.142	0.980	2.142	0.980	2.142	0.980	2.142	0.980	2.142	0.980
41	2.126	0.980	2.126	0.980	2.126	0.980	2.126	0.980	2.126	0.980
$s_{r,n}$	2.111	0.980	2.111	0.980	2.111	0.980	2.111	0.980	2.111	0.980
true min availability	0.980		0.980		0.980		0.980		0.980	
true average cost	5.597		5.597		5.597		5.597		5.597	
efficiency	1.000		1.000		1.000		1.000		1.000	

Table 13. *The minimum average cost and the optimal IRR policy as I_i varies for $\Gamma(\alpha,\beta)$.*

I_i	2.5(1+0.02(i-1))		3.75(1+0.02(i-1))		5(1+0.02(i-1))*		6.25(1+0.02(i-1))		7.5(1+0.02(i-1))	
availability	0.986		0.982		0.980		0.978		0.974	
average cost	4.254		4.961		5.597*		6.183		6.731	
number of inspection	55		46		41		37		34	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	2.787	0.986	2.975	0.982	3.059	0.980	3.138	0.978	3.282	0.974
2	2.687	0.986	2.867	0.982	2.947	0.980	3.022	0.978	3.160	0.974
3	2.660	0.986	2.839	0.982	2.918	0.980	2.993	0.978	3.129	0.974
4	2.635	0.986	2.811	0.982	2.890	0.980	2.964	0.978	3.099	0.974
5	2.609	0.986	2.784	0.982	2.862	0.980	2.935	0.978	3.069	0.974
6	2.584	0.986	2.758	0.982	2.835	0.980	2.907	0.978	3.040	0.974
7	2.560	0.986	2.732	0.982	2.808	0.980	2.880	0.978	3.011	0.974
8	2.536	0.986	2.706	0.982	2.782	0.980	2.853	0.978	2.983	0.974
9	2.513	0.986	2.681	0.982	2.756	0.980	2.827	0.978	2.956	0.974
10	2.490	0.986	2.657	0.982	2.731	0.980	2.801	0.978	2.928	0.974
11	2.467	0.986	2.633	0.982	2.706	0.980	2.775	0.978	2.902	0.974
12	2.445	0.986	2.609	0.982	2.682	0.980	2.750	0.978	2.876	0.974
13	2.423	0.986	2.586	0.982	2.658	0.980	2.726	0.978	2.850	0.974
14	2.402	0.986	2.563	0.982	2.634	0.980	2.702	0.978	2.825	0.974
15	2.380	0.986	2.540	0.982	2.611	0.980	2.678	0.978	2.800	0.974
16	2.360	0.986	2.518	0.982	2.589	0.980	2.655	0.978	2.776	0.974
17	2.339	0.986	2.496	0.982	2.566	0.980	2.632	0.978	2.752	0.974
18	2.319	0.986	2.475	0.982	2.544	0.980	2.609	0.978	2.728	0.974
19	2.300	0.986	2.454	0.982	2.523	0.980	2.587	0.978	2.705	0.974
20	2.280	0.986	2.433	0.982	2.502	0.980	2.565	0.978	2.682	0.974
21	2.261	0.986	2.413	0.982	2.481	0.980	2.544	0.978	2.660	0.974
22	2.243	0.986	2.393	0.982	2.460	0.980	2.523	0.978	2.638	0.974
23	2.224	0.986	2.374	0.982	2.440	0.980	2.502	0.978	2.616	0.974
24	2.206	0.986	2.354	0.982	2.420	0.980	2.482	0.978	2.595	0.974
25	2.188	0.986	2.335	0.982	2.401	0.980	2.462	0.978	2.574	0.974
26	2.171	0.986	2.317	0.982	2.381	0.980	2.442	0.978	2.554	0.974
27	2.154	0.986	2.298	0.982	2.363	0.980	2.423	0.978	2.533	0.974
28	2.137	0.986	2.280	0.982	2.344	0.980	2.404	0.978	2.513	0.974
29	2.120	0.986	2.262	0.982	2.326	0.980	2.385	0.978	2.494	0.974
30	2.104	0.986	2.245	0.982	2.308	0.980	2.366	0.978	2.474	0.974
31	2.087	0.986	2.228	0.982	2.290	0.980	2.348	0.978	2.455	0.974
32	2.072	0.986	2.211	0.982	2.272	0.980	2.330	0.978	2.437	0.974
33	2.056	0.986	2.194	0.982	2.255	0.980	2.313	0.978	2.418	0.974
34	2.040	0.986	2.177	0.982	2.238	0.980	2.295	0.978	2.400	0.974
35	2.025	0.986	2.161	0.982	2.222	0.980	2.278	0.978		
36	2.010	0.986	2.145	0.982	2.205	0.980	2.261	0.978		
37	1.995	0.986	2.129	0.982	2.189	0.980	2.245	0.978		
38	1.981	0.986	2.114	0.982	2.173	0.980				
39	1.966	0.986	2.098	0.982	2.157	0.980				
40	1.952	0.986	2.083	0.982	2.142	0.980				
41	1.938	0.986	2.068	0.982	2.126	0.980				
42	1.925	0.986	2.054	0.982						
43	1.911	0.986	2.039	0.982						
44	1.898	0.986	2.025	0.982						
45	1.885	0.986	2.011	0.982						
46	1.872	0.986	1.997	0.982						
47	1.859	0.986								
48	1.846	0.986								
49	1.834	0.986								
50	1.821	0.986								
51	1.809	0.986								

Table 13. *(Continue)*

52	1.797	0.986								
53	1.785	0.986								
54	1.774	0.986								
55	1.762	0.986								
$s_{r,n}$	1.751	0.986	1.983	0.982	2.111	0.980	2.228	0.978	2.382	0.974
true min availability	0.986		0.982		0.980		0.978		0.974	
true average cost	5.798		5.623		5.597		5.614		5.677	
efficiency	0.965		0.995		1.000		0.997		0.986	

Table 14. *The minimum average cost and the optimal IRR policy as C_i varies for $\Gamma(\alpha,\beta)$.*

C_i	4(1+0.02(i-1))		6(1+0.02(i-1))		8(1+0.02(i-1))*		10(1+0.02(i-1))		12(1+0.02(i-1))	
availability	0.980		0.980		0.980		0.980		0.980	
average cost	5.367		5.483		5.597*		5.710		5.822	
number of inspection	43		42		41		40		40	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	3.059	0.980	3.059	0.980	3.059	0.980	3.059	0.980	3.059	0.980
2	2.947	0.980	2.947	0.980	2.947	0.980	2.947	0.980	2.947	0.980
3	2.918	0.980	2.918	0.980	2.918	0.980	2.918	0.980	2.918	0.980
4	2.890	0.980	2.890	0.980	2.890	0.980	2.890	0.980	2.890	0.980
5	2.862	0.980	2.862	0.980	2.862	0.980	2.862	0.980	2.862	0.980
6	2.835	0.980	2.835	0.980	2.835	0.980	2.835	0.980	2.835	0.980
7	2.808	0.980	2.808	0.980	2.808	0.980	2.808	0.980	2.808	0.980
8	2.782	0.980	2.782	0.980	2.782	0.980	2.782	0.980	2.782	0.980
9	2.756	0.980	2.756	0.980	2.756	0.980	2.756	0.980	2.756	0.980
10	2.731	0.980	2.731	0.980	2.731	0.980	2.731	0.980	2.731	0.980
11	2.706	0.980	2.706	0.980	2.706	0.980	2.706	0.980	2.706	0.980
12	2.682	0.980	2.682	0.980	2.682	0.980	2.682	0.980	2.682	0.980
13	2.658	0.980	2.658	0.980	2.658	0.980	2.658	0.980	2.658	0.980
14	2.634	0.980	2.634	0.980	2.634	0.980	2.634	0.980	2.634	0.980
15	2.611	0.980	2.611	0.980	2.611	0.980	2.611	0.980	2.611	0.980
16	2.589	0.980	2.589	0.980	2.589	0.980	2.589	0.980	2.589	0.980
17	2.566	0.980	2.566	0.980	2.566	0.980	2.566	0.980	2.566	0.980
18	2.544	0.980	2.544	0.980	2.544	0.980	2.544	0.980	2.544	0.980
19	2.523	0.980	2.523	0.980	2.523	0.980	2.523	0.980	2.523	0.980
20	2.502	0.980	2.502	0.980	2.502	0.980	2.502	0.980	2.502	0.980
21	2.481	0.980	2.481	0.980	2.481	0.980	2.481	0.980	2.481	0.980
22	2.460	0.980	2.460	0.980	2.460	0.980	2.460	0.980	2.460	0.980
23	2.440	0.980	2.440	0.980	2.440	0.980	2.440	0.980	2.440	0.980
24	2.420	0.980	2.420	0.980	2.420	0.980	2.420	0.980	2.420	0.980
25	2.401	0.980	2.401	0.980	2.401	0.980	2.401	0.980	2.401	0.980
26	2.381	0.980	2.381	0.980	2.381	0.980	2.381	0.980	2.381	0.980
27	2.363	0.980	2.363	0.980	2.363	0.980	2.363	0.980	2.363	0.980
28	2.344	0.980	2.344	0.980	2.344	0.980	2.344	0.980	2.344	0.980
29	2.326	0.980	2.326	0.980	2.326	0.980	2.326	0.980	2.326	0.980
30	2.308	0.980	2.308	0.980	2.308	0.980	2.308	0.980	2.308	0.980
31	2.290	0.980	2.290	0.980	2.290	0.980	2.290	0.980	2.290	0.980
32	2.272	0.980	2.272	0.980	2.272	0.980	2.272	0.980	2.272	0.980
33	2.255	0.980	2.255	0.980	2.255	0.980	2.255	0.980	2.255	0.980
34	2.238	0.980	2.238	0.980	2.238	0.980	2.238	0.980	2.238	0.980
35	2.222	0.980	2.222	0.980	2.222	0.980	2.222	0.980	2.222	0.980
36	2.205	0.980	2.205	0.980	2.205	0.980	2.205	0.980	2.205	0.980
37	2.189	0.980	2.189	0.980	2.189	0.980	2.189	0.980	2.189	0.980
38	2.173	0.980	2.173	0.980	2.173	0.980	2.173	0.980	2.173	0.980
39	2.157	0.980	2.157	0.980	2.157	0.980	2.157	0.980	2.157	0.980
40	2.142	0.980	2.142	0.980	2.142	0.980	2.142	0.980	2.142	0.980
41	2.126	0.980	2.126	0.980	2.126	0.980				
42	2.111	0.980	2.111	0.980						
43	2.096	0.980								
$s_{r,n}$	2.082	0.980	2.096	0.980	2.111	0.980	2.126	0.980	2.126	0.980
true min availability	0.980		0.980		0.980		0.980		0.980	
true average cost	5.599		5.597		5.597		5.598		5.598	
efficiency	1.000		1.000		1.000		1.000		1.000	

Table 15. *The minimum average cost and the optimal IRR policy as γ varies for exp(γ).*

γ	0.5		0.75		1*		1.25		1.5	
availability	0.960		0.954		0.946		0.942		0.936	
average cost	47.464		59.004		68.972*		77.922		86.179	
number of inspection	38		38		38		38		38	
1	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
2	8.16E-02	0.922	6.28E-02	0.939	5.55E-02	0.946	4.78E-02	0.953	4.41E-02	0.957
3	7.18E-02	0.922	5.53E-02	0.939	4.89E-02	0.946	4.21E-02	0.953	3.88E-02	0.957
4	7.01E-02	0.922	5.39E-02	0.939	4.77E-02	0.946	4.11E-02	0.953	3.79E-02	0.957
5	6.84E-02	0.922	5.26E-02	0.939	4.65E-02	0.946	4.01E-02	0.953	3.70E-02	0.957
6	6.67E-02	0.922	5.13E-02	0.939	4.54E-02	0.946	3.91E-02	0.953	3.61E-02	0.957
7	6.51E-02	0.922	5.01E-02	0.939	4.43E-02	0.946	3.81E-02	0.953	3.52E-02	0.957
8	6.35E-02	0.922	4.88E-02	0.939	4.32E-02	0.946	3.72E-02	0.953	3.43E-02	0.957
9	6.19E-02	0.922	4.77E-02	0.939	4.21E-02	0.946	3.63E-02	0.953	3.35E-02	0.957
10	6.04E-02	0.922	4.65E-02	0.939	4.11E-02	0.946	3.54E-02	0.953	3.27E-02	0.957
11	5.90E-02	0.922	4.54E-02	0.939	4.01E-02	0.946	3.45E-02	0.953	3.19E-02	0.957
12	5.75E-02	0.922	4.42E-02	0.939	3.91E-02	0.946	3.37E-02	0.953	3.11E-02	0.957
13	5.61E-02	0.922	4.32E-02	0.939	3.82E-02	0.946	3.29E-02	0.953	3.03E-02	0.957
14	5.47E-02	0.922	4.21E-02	0.939	3.73E-02	0.946	3.21E-02	0.953	2.96E-02	0.957
15	5.34E-02	0.922	4.11E-02	0.939	3.63E-02	0.946	3.13E-02	0.953	2.89E-02	0.957
16	5.21E-02	0.922	4.01E-02	0.939	3.55E-02	0.946	3.05E-02	0.953	2.82E-02	0.957
17	5.08E-02	0.922	3.91E-02	0.939	3.46E-02	0.946	2.98E-02	0.953	2.75E-02	0.957
18	4.96E-02	0.922	3.82E-02	0.939	3.37E-02	0.946	2.91E-02	0.953	2.68E-02	0.957
19	4.84E-02	0.922	3.72E-02	0.939	3.29E-02	0.946	2.84E-02	0.953	2.62E-02	0.957
20	4.72E-02	0.922	3.63E-02	0.939	3.21E-02	0.946	2.77E-02	0.953	2.55E-02	0.957
21	4.61E-02	0.922	3.54E-02	0.939	3.13E-02	0.946	2.70E-02	0.953	2.49E-02	0.957
22	4.49E-02	0.922	3.46E-02	0.939	3.06E-02	0.946	2.63E-02	0.953	2.43E-02	0.957
23	4.38E-02	0.922	3.37E-02	0.939	2.98E-02	0.946	2.57E-02	0.953	2.37E-02	0.957
24	4.28E-02	0.922	3.29E-02	0.939	2.91E-02	0.946	2.51E-02	0.953	2.31E-02	0.957
25	4.17E-02	0.922	3.21E-02	0.939	2.84E-02	0.946	2.45E-02	0.953	2.26E-02	0.957
26	4.07E-02	0.922	3.13E-02	0.939	2.77E-02	0.946	2.39E-02	0.953	2.20E-02	0.957
27	3.97E-02	0.922	3.06E-02	0.939	2.70E-02	0.946	2.33E-02	0.953	2.15E-02	0.957
28	3.87E-02	0.922	2.98E-02	0.939	2.64E-02	0.946	2.27E-02	0.953	2.10E-02	0.957
29	3.78E-02	0.922	2.91E-02	0.939	2.57E-02	0.946	2.22E-02	0.953	2.04E-02	0.957
30	3.69E-02	0.922	2.84E-02	0.939	2.51E-02	0.946	2.16E-02	0.953	1.99E-02	0.957
31	3.60E-02	0.922	2.77E-02	0.939	2.45E-02	0.946	2.11E-02	0.953	1.95E-02	0.957
32	3.51E-02	0.922	2.70E-02	0.939	2.39E-02	0.946	2.06E-02	0.953	1.90E-02	0.957
33	3.42E-02	0.922	2.63E-02	0.939	2.33E-02	0.946	2.01E-02	0.953	1.85E-02	0.957
34	3.34E-02	0.922	2.57E-02	0.939	2.27E-02	0.946	1.96E-02	0.953	1.81E-02	0.957
35	3.26E-02	0.922	2.51E-02	0.939	2.22E-02	0.946	1.91E-02	0.953	1.76E-02	0.957
36	3.18E-02	0.922	2.45E-02	0.939	2.16E-02	0.946	1.86E-02	0.953	1.72E-02	0.957
37	3.10E-02	0.922	2.39E-02	0.939	2.11E-02	0.946	1.82E-02	0.953	1.68E-02	0.957
38	3.03E-02	0.922	2.33E-02	0.939	2.06E-02	0.946	1.77E-02	0.953	1.64E-02	0.957
39	2.95E-02	0.922	2.27E-02	0.939	2.01E-02	0.946	1.73E-02	0.953	1.60E-02	0.957
$s_{r,n}$	2.88E-02	0.922	2.22E-02	0.939	1.96E-02	0.946	1.69E-02	0.953	1.56E-02	0.957
true min availability	0.922		0.939		0.946		0.953		0.957	
true average cost	76.165		69.805		68.972		69.651		70.743	
efficiency	0.906		0.988		1.000		0.990		0.975	

Table 16. *The minimum average cost and the optimal IRR policy as a varies for exp(γ).*

a	1.0130		1.0190		1.025*		1.0375		1.0500	
availability	0.952		0.950		0.946		0.942		0.940	
average cost	60.885		65.240		68.972*		75.536		81.068	
number of inspection	56		45		38		30		25	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	4.92E-02	0.952	5.13E-02	0.950	5.55E-02	0.946	5.98E-02	0.942	6.19E-02	0.940
2	4.38E-02	0.951	4.54E-02	0.950	4.89E-02	0.946	5.20E-02	0.943	5.32E-02	0.941
3	4.32E-02	0.951	4.46E-02	0.949	4.77E-02	0.946	5.01E-02	0.943	5.07E-02	0.943
4	4.27E-02	0.950	4.37E-02	0.949	4.65E-02	0.946	4.83E-02	0.944	4.83E-02	0.944
5	4.21E-02	0.950	4.29E-02	0.949	4.54E-02	0.946	4.65E-02	0.945	4.60E-02	0.945
6	4.16E-02	0.949	4.21E-02	0.949	4.43E-02	0.946	4.49E-02	0.945	4.38E-02	0.947
7	4.11E-02	0.949	4.13E-02	0.948	4.32E-02	0.946	4.32E-02	0.946	4.17E-02	0.948
8	4.05E-02	0.948	4.06E-02	0.948	4.21E-02	0.946	4.17E-02	0.947	3.97E-02	0.949
9	4.00E-02	0.947	3.98E-02	0.948	4.11E-02	0.946	4.02E-02	0.947	3.78E-02	0.950
10	3.95E-02	0.947	3.91E-02	0.947	4.01E-02	0.946	3.87E-02	0.948	3.60E-02	0.951
11	3.90E-02	0.946	3.83E-02	0.947	3.91E-02	0.946	3.73E-02	0.948	3.43E-02	0.953
12	3.85E-02	0.946	3.76E-02	0.947	3.82E-02	0.946	3.60E-02	0.949	3.27E-02	0.954
13	3.80E-02	0.945	3.69E-02	0.946	3.73E-02	0.946	3.47E-02	0.950	3.11E-02	0.955
14	3.75E-02	0.944	3.62E-02	0.946	3.63E-02	0.946	3.34E-02	0.950	2.96E-02	0.956
15	3.70E-02	0.944	3.56E-02	0.946	3.55E-02	0.946	3.22E-02	0.951	2.82E-02	0.957
16	3.66E-02	0.943	3.49E-02	0.946	3.46E-02	0.946	3.10E-02	0.951	2.69E-02	0.958
17	3.61E-02	0.942	3.42E-02	0.945	3.37E-02	0.946	2.99E-02	0.952	2.56E-02	0.959
18	3.56E-02	0.942	3.36E-02	0.945	3.29E-02	0.946	2.88E-02	0.953	2.44E-02	0.960
19	3.52E-02	0.941	3.30E-02	0.945	3.21E-02	0.946	2.78E-02	0.953	2.32E-02	0.961
20	3.47E-02	0.940	3.24E-02	0.944	3.13E-02	0.946	2.68E-02	0.954	2.21E-02	0.962
21	3.43E-02	0.940	3.18E-02	0.944	3.06E-02	0.946	2.58E-02	0.954	2.11E-02	0.962
22	3.38E-02	0.939	3.12E-02	0.944	2.98E-02	0.946	2.49E-02	0.955	2.00E-02	0.963
23	3.34E-02	0.938	3.06E-02	0.943	2.91E-02	0.946	2.40E-02	0.955	1.91E-02	0.964
24	3.30E-02	0.938	3.00E-02	0.943	2.84E-02	0.946	2.31E-02	0.956	1.82E-02	0.965
25	3.25E-02	0.937	2.95E-02	0.943	2.77E-02	0.946	2.23E-02	0.956	1.73E-02	0.966
26	3.21E-02	0.936	2.89E-02	0.942	2.70E-02	0.946	2.15E-02	0.957		
27	3.17E-02	0.935	2.84E-02	0.942	2.64E-02	0.946	2.07E-02	0.957		
28	3.13E-02	0.935	2.78E-02	0.942	2.57E-02	0.946	2.00E-02	0.958		
29	3.09E-02	0.934	2.73E-02	0.941	2.51E-02	0.946	1.92E-02	0.958		
30	3.05E-02	0.933	2.68E-02	0.941	2.45E-02	0.946	1.85E-02	0.959		
31	3.01E-02	0.932	2.63E-02	0.941	2.39E-02	0.946				
32	2.97E-02	0.932	2.58E-02	0.940	2.33E-02	0.946				
33	2.94E-02	0.931	2.53E-02	0.940	2.27E-02	0.946				
34	2.90E-02	0.930	2.49E-02	0.940	2.22E-02	0.946				
35	2.86E-02	0.929	2.44E-02	0.939	2.16E-02	0.946				
36	2.82E-02	0.928	2.40E-02	0.939	2.11E-02	0.946				
37	2.79E-02	0.928	2.35E-02	0.939	2.06E-02	0.946				
38	2.75E-02	0.927	2.31E-02	0.938	2.01E-02	0.946				
39	2.72E-02	0.926	2.26E-02	0.938						
40	2.68E-02	0.925	2.22E-02	0.938						
41	2.65E-02	0.924	2.18E-02	0.937						
42	2.61E-02	0.923	2.14E-02	0.937						
43	2.58E-02	0.923	2.10E-02	0.937						
44	2.55E-02	0.922	2.06E-02	0.936						
45	2.51E-02	0.921	2.02E-02	0.936						
46	2.48E-02	0.920								
47	2.45E-02	0.919								
48	2.42E-02	0.918								

Table 16. *(Continue)*

49	2.39E-02	0.917								
50	2.36E-02	0.916								
51	2.33E-02	0.915								
52	2.30E-02	0.914								
53	2.27E-02	0.913								
54	2.24E-02	0.912								
55	2.21E-02	0.911								
56	2.18E-02	0.910								
$s_{r,n}$	2.15E-02	0.909	1.98E-02	0.935	1.96E-02	0.946	1.79E-02	0.959	1.65E-02	0.967
true min availability	0.909		0.935		0.946		0.942		0.940	
true average cost	94.431		75.882		68.972		75.225		80.572	
efficiency	0.730		0.909		1.000		0.917		0.856	

Table 17. *The minimum average cost and the optimal IRR policy as p varies for exp(γ).*

p	0.8		0.85		0.9*		0.95		1	
availability	0.944		0.946		0.946		0.948		0.950	
average cost	72.950		70.869		68.971*		67.202		65.587	
number of inspection	38		38		38		38		38	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	5.76E-02	0.944	5.55E-02	0.946	5.55E-02	0.946	5.34E-02	0.948	5.13E-02	0.950
2	4.52E-02	0.950	4.62E-02	0.949	4.89E-02	0.946	4.96E-02	0.945	5.00E-02	0.945
3	4.41E-02	0.950	4.51E-02	0.949	4.77E-02	0.946	4.83E-02	0.945	4.88E-02	0.945
4	4.31E-02	0.950	4.40E-02	0.949	4.65E-02	0.946	4.72E-02	0.945	4.76E-02	0.945
5	4.20E-02	0.950	4.29E-02	0.949	4.54E-02	0.946	4.60E-02	0.945	4.65E-02	0.945
6	4.10E-02	0.950	4.19E-02	0.949	4.43E-02	0.946	4.49E-02	0.945	4.53E-02	0.945
7	4.00E-02	0.950	4.09E-02	0.949	4.32E-02	0.946	4.38E-02	0.945	4.42E-02	0.945
8	3.90E-02	0.950	3.99E-02	0.949	4.21E-02	0.946	4.27E-02	0.945	4.32E-02	0.945
9	3.81E-02	0.950	3.89E-02	0.949	4.11E-02	0.946	4.17E-02	0.945	4.21E-02	0.945
10	3.71E-02	0.950	3.79E-02	0.949	4.01E-02	0.946	4.07E-02	0.945	4.11E-02	0.945
11	3.62E-02	0.950	3.70E-02	0.949	3.91E-02	0.946	3.97E-02	0.945	4.01E-02	0.945
12	3.53E-02	0.950	3.61E-02	0.949	3.82E-02	0.946	3.87E-02	0.945	3.91E-02	0.945
13	3.45E-02	0.950	3.52E-02	0.949	3.73E-02	0.946	3.78E-02	0.945	3.81E-02	0.945
14	3.36E-02	0.950	3.44E-02	0.949	3.63E-02	0.946	3.68E-02	0.945	3.72E-02	0.945
15	3.28E-02	0.950	3.35E-02	0.949	3.55E-02	0.946	3.60E-02	0.945	3.63E-02	0.945
16	3.20E-02	0.950	3.27E-02	0.949	3.46E-02	0.946	3.51E-02	0.945	3.54E-02	0.945
17	3.12E-02	0.950	3.19E-02	0.949	3.37E-02	0.946	3.42E-02	0.945	3.46E-02	0.945
18	3.05E-02	0.950	3.11E-02	0.949	3.29E-02	0.946	3.34E-02	0.945	3.37E-02	0.945
19	2.97E-02	0.950	3.04E-02	0.949	3.21E-02	0.946	3.26E-02	0.945	3.29E-02	0.945
20	2.90E-02	0.950	2.96E-02	0.949	3.13E-02	0.946	3.18E-02	0.945	3.21E-02	0.945
21	2.83E-02	0.950	2.89E-02	0.949	3.06E-02	0.946	3.10E-02	0.945	3.13E-02	0.945
22	2.76E-02	0.950	2.82E-02	0.949	2.98E-02	0.946	3.02E-02	0.945	3.05E-02	0.945
23	2.69E-02	0.950	2.75E-02	0.949	2.91E-02	0.946	2.95E-02	0.945	2.98E-02	0.945
24	2.63E-02	0.950	2.69E-02	0.949	2.84E-02	0.946	2.88E-02	0.945	2.91E-02	0.945
25	2.56E-02	0.950	2.62E-02	0.949	2.77E-02	0.946	2.81E-02	0.945	2.84E-02	0.945
26	2.50E-02	0.950	2.56E-02	0.949	2.70E-02	0.946	2.74E-02	0.945	2.77E-02	0.945
27	2.44E-02	0.950	2.49E-02	0.949	2.64E-02	0.946	2.67E-02	0.945	2.70E-02	0.945
28	2.38E-02	0.950	2.43E-02	0.949	2.57E-02	0.946	2.61E-02	0.945	2.63E-02	0.945
29	2.32E-02	0.950	2.37E-02	0.949	2.51E-02	0.946	2.54E-02	0.945	2.57E-02	0.945
30	2.27E-02	0.950	2.32E-02	0.949	2.45E-02	0.946	2.48E-02	0.945	2.51E-02	0.945
31	2.21E-02	0.950	2.26E-02	0.949	2.39E-02	0.946	2.42E-02	0.945	2.45E-02	0.945
32	2.16E-02	0.950	2.20E-02	0.949	2.33E-02	0.946	2.36E-02	0.945	2.39E-02	0.945
33	2.10E-02	0.950	2.15E-02	0.949	2.27E-02	0.946	2.31E-02	0.945	2.33E-02	0.945
34	2.05E-02	0.950	2.10E-02	0.949	2.22E-02	0.946	2.25E-02	0.945	2.27E-02	0.945
35	2.00E-02	0.950	2.05E-02	0.949	2.16E-02	0.946	2.19E-02	0.945	2.22E-02	0.945
36	1.95E-02	0.950	2.00E-02	0.949	2.11E-02	0.946	2.14E-02	0.945	2.16E-02	0.945
37	1.91E-02	0.950	1.95E-02	0.949	2.06E-02	0.946	2.09E-02	0.945	2.11E-02	0.945
38	1.86E-02	0.950	1.90E-02	0.949	2.01E-02	0.946	2.04E-02	0.945	2.06E-02	0.945
$s_{r,n}$	1.81E-02	0.950	1.85E-02	0.949	1.96E-02	0.946	1.99E-02	0.945	2.01E-02	0.945
true min availability	0.944		0.946		0.946		0.945269		0.944751	
true average cost	72.998		70.891		68.972		69.078		69.177	
efficiency	0.945		0.973		1.000		0.998		0.997	

Table 18. *The minimum average cost and the optimal IRR policy as p' varies for exp(γ).*

p'	0.8		0.85		0.9*		0.95		1	
availability	0.944		0.946		0.946		0.948		0.950	
average cost	73.077		71.070		68.972*		66.749		64.408	
number of inspection	35		36		38		40		42	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	5.76E-02	0.944	5.55E-02	0.946	5.55E-02	0.946	5.34E-02	0.948	5.13E-02	0.95
2	5.07E-02	0.944	4.89E-02	0.946	4.89E-02	0.946	4.70E-02	0.948	4.52E-02	0.95
3	4.95E-02	0.944	4.77E-02	0.946	4.77E-02	0.946	4.59E-02	0.948	4.41E-02	0.95
4	4.83E-02	0.944	4.65E-02	0.946	4.65E-02	0.946	4.47E-02	0.948	4.30E-02	0.95
5	4.71E-02	0.944	4.54E-02	0.946	4.54E-02	0.946	4.37E-02	0.948	4.19E-02	0.95
6	4.60E-02	0.944	4.43E-02	0.946	4.43E-02	0.946	4.26E-02	0.948	4.09E-02	0.95
7	4.49E-02	0.944	4.32E-02	0.946	4.32E-02	0.946	4.16E-02	0.948	3.99E-02	0.95
8	4.38E-02	0.944	4.21E-02	0.946	4.21E-02	0.946	4.05E-02	0.948	3.89E-02	0.95
9	4.27E-02	0.944	4.11E-02	0.946	4.11E-02	0.946	3.95E-02	0.948	3.80E-02	0.95
10	4.16E-02	0.944	4.01E-02	0.946	4.01E-02	0.946	3.86E-02	0.948	3.71E-02	0.95
11	4.06E-02	0.944	3.91E-02	0.946	3.91E-02	0.946	3.76E-02	0.948	3.62E-02	0.95
12	3.96E-02	0.944	3.82E-02	0.946	3.82E-02	0.946	3.67E-02	0.948	3.53E-02	0.95
13	3.87E-02	0.944	3.73E-02	0.946	3.73E-02	0.946	3.58E-02	0.948	3.44E-02	0.95
14	3.77E-02	0.944	3.63E-02	0.946	3.63E-02	0.946	3.50E-02	0.948	3.36E-02	0.95
15	3.68E-02	0.944	3.55E-02	0.946	3.55E-02	0.946	3.41E-02	0.948	3.28E-02	0.95
16	3.59E-02	0.944	3.46E-02	0.946	3.46E-02	0.946	3.33E-02	0.948	3.20E-02	0.95
17	3.50E-02	0.944	3.37E-02	0.946	3.37E-02	0.946	3.25E-02	0.948	3.12E-02	0.95
18	3.42E-02	0.944	3.29E-02	0.946	3.29E-02	0.946	3.17E-02	0.948	3.04E-02	0.95
19	3.33E-02	0.944	3.21E-02	0.946	3.21E-02	0.946	3.09E-02	0.948	2.97E-02	0.95
20	3.25E-02	0.944	3.13E-02	0.946	3.13E-02	0.946	3.01E-02	0.948	2.89E-02	0.95
21	3.17E-02	0.944	3.06E-02	0.946	3.06E-02	0.946	2.94E-02	0.948	2.82E-02	0.95
22	3.10E-02	0.944	2.98E-02	0.946	2.98E-02	0.946	2.87E-02	0.948	2.76E-02	0.95
23	3.02E-02	0.944	2.91E-02	0.946	2.91E-02	0.946	2.80E-02	0.948	2.69E-02	0.95
24	2.95E-02	0.944	2.84E-02	0.946	2.84E-02	0.946	2.73E-02	0.948	2.62E-02	0.95
25	2.88E-02	0.944	2.77E-02	0.946	2.77E-02	0.946	2.66E-02	0.948	2.56E-02	0.95
26	2.81E-02	0.944	2.70E-02	0.946	2.70E-02	0.946	2.60E-02	0.948	2.50E-02	0.95
27	2.74E-02	0.944	2.64E-02	0.946	2.64E-02	0.946	2.54E-02	0.948	2.44E-02	0.95
28	2.67E-02	0.944	2.57E-02	0.946	2.57E-02	0.946	2.47E-02	0.948	2.38E-02	0.95
29	2.61E-02	0.944	2.51E-02	0.946	2.51E-02	0.946	2.41E-02	0.948	2.32E-02	0.95
30	2.54E-02	0.944	2.45E-02	0.946	2.45E-02	0.946	2.35E-02	0.948	2.26E-02	0.95
31	2.48E-02	0.944	2.39E-02	0.946	2.39E-02	0.946	2.30E-02	0.948	2.21E-02	0.95
32	2.42E-02	0.944	2.33E-02	0.946	2.33E-02	0.946	2.24E-02	0.948	2.15E-02	0.95
33	2.36E-02	0.944	2.27E-02	0.946	2.27E-02	0.946	2.19E-02	0.948	2.10E-02	0.95
34	2.30E-02	0.944	2.22E-02	0.946	2.22E-02	0.946	2.13E-02	0.948	2.05E-02	0.95
35	2.25E-02	0.944	2.16E-02	0.946	2.16E-02	0.946	2.08E-02	0.948	2.00E-02	0.95
36			2.11E-02	0.946	2.11E-02	0.946	2.03E-02	0.948	1.95E-02	0.95
37					2.06E-02	0.946	1.98E-02	0.948	1.90E-02	0.95
38					2.01E-02	0.946	1.93E-02	0.948	1.86E-02	0.95
39							1.89E-02	0.948	1.81E-02	0.95
40							1.84E-02	0.948	1.77E-02	0.95
41									1.72E-02	0.95
42									1.68E-02	0.95
$s_{r,n}$	2.19E-02	0.944	2.06E-02	0.946	1.96E-02	0.946	1.79E-02	0.948	1.64E-02	0.95
true min availability	0.944		0.946		0.946		0.948		0.95	
true average cost	69.148		69.003		68.972		69.001		69.210	
efficiency	0.997		1.000		1.000		1.000		0.997	

Table 19. *The minimum average cost and the optimal IRR policy as I_l varies for exp(γ).*

I_l	0.25		0.375		0.5*		0.625		0.75	
availability	0.952		0.950		0.946		0.944		0.942	
average cost	60.612		64.998		68.972*		72.616		76.018	
number of inspection	46		42		38		35		33	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	4.92E-02	0.952	5.13E-02	0.950	5.55E-02	0.946	5.76E-02	0.944	5.98E-02	0.942
2	4.33E-02	0.952	4.52E-02	0.950	4.89E-02	0.946	5.07E-02	0.944	5.26E-02	0.942
3	4.22E-02	0.952	4.41E-02	0.950	4.77E-02	0.946	4.95E-02	0.944	5.13E-02	0.942
4	4.12E-02	0.952	4.30E-02	0.950	4.65E-02	0.946	4.83E-02	0.944	5.01E-02	0.942
5	4.02E-02	0.952	4.19E-02	0.950	4.54E-02	0.946	4.71E-02	0.944	4.89E-02	0.942
6	3.92E-02	0.952	4.09E-02	0.950	4.43E-02	0.946	4.60E-02	0.944	4.77E-02	0.942
7	3.83E-02	0.952	3.99E-02	0.950	4.32E-02	0.946	4.49E-02	0.944	4.65E-02	0.942
8	3.73E-02	0.952	3.89E-02	0.950	4.21E-02	0.946	4.38E-02	0.944	4.54E-02	0.942
9	3.64E-02	0.952	3.80E-02	0.950	4.11E-02	0.946	4.27E-02	0.944	4.43E-02	0.942
10	3.55E-02	0.952	3.71E-02	0.950	4.01E-02	0.946	4.16E-02	0.944	4.32E-02	0.942
11	3.47E-02	0.952	3.62E-02	0.950	3.91E-02	0.946	4.06E-02	0.944	4.21E-02	0.942
12	3.38E-02	0.952	3.53E-02	0.950	3.82E-02	0.946	3.96E-02	0.944	4.11E-02	0.942
13	3.30E-02	0.952	3.44E-02	0.950	3.73E-02	0.946	3.87E-02	0.944	4.01E-02	0.942
14	3.22E-02	0.952	3.36E-02	0.950	3.63E-02	0.946	3.77E-02	0.944	3.91E-02	0.942
15	3.14E-02	0.952	3.28E-02	0.950	3.55E-02	0.946	3.68E-02	0.944	3.82E-02	0.942
16	3.06E-02	0.952	3.20E-02	0.950	3.46E-02	0.946	3.59E-02	0.944	3.72E-02	0.942
17	2.99E-02	0.952	3.12E-02	0.950	3.37E-02	0.946	3.50E-02	0.944	3.63E-02	0.942
18	2.92E-02	0.952	3.04E-02	0.950	3.29E-02	0.946	3.42E-02	0.944	3.54E-02	0.942
19	2.85E-02	0.952	2.97E-02	0.950	3.21E-02	0.946	3.33E-02	0.944	3.46E-02	0.942
20	2.78E-02	0.952	2.89E-02	0.950	3.13E-02	0.946	3.25E-02	0.944	3.37E-02	0.942
21	2.71E-02	0.952	2.82E-02	0.950	3.06E-02	0.946	3.17E-02	0.944	3.29E-02	0.942
22	2.64E-02	0.952	2.76E-02	0.950	2.98E-02	0.946	3.10E-02	0.944	3.21E-02	0.942
23	2.58E-02	0.952	2.69E-02	0.950	2.91E-02	0.946	3.02E-02	0.944	3.13E-02	0.942
24	2.51E-02	0.952	2.62E-02	0.950	2.84E-02	0.946	2.95E-02	0.944	3.06E-02	0.942
25	2.45E-02	0.952	2.56E-02	0.950	2.77E-02	0.946	2.88E-02	0.944	2.98E-02	0.942
26	2.39E-02	0.952	2.50E-02	0.950	2.70E-02	0.946	2.81E-02	0.944	2.91E-02	0.942
27	2.34E-02	0.952	2.44E-02	0.950	2.64E-02	0.946	2.74E-02	0.944	2.84E-02	0.942
28	2.28E-02	0.952	2.38E-02	0.950	2.57E-02	0.946	2.67E-02	0.944	2.77E-02	0.942
29	2.22E-02	0.952	2.32E-02	0.950	2.51E-02	0.946	2.61E-02	0.944	2.70E-02	0.942
30	2.17E-02	0.952	2.26E-02	0.950	2.45E-02	0.946	2.54E-02	0.944	2.64E-02	0.942
31	2.12E-02	0.952	2.21E-02	0.950	2.39E-02	0.946	2.48E-02	0.944	2.57E-02	0.942
32	2.06E-02	0.952	2.15E-02	0.950	2.33E-02	0.946	2.42E-02	0.944	2.51E-02	0.942
33	2.01E-02	0.952	2.10E-02	0.950	2.27E-02	0.946	2.36E-02	0.944	2.45E-02	0.942
34	1.96E-02	0.952	2.05E-02	0.950	2.22E-02	0.946	2.30E-02	0.944		
35	1.92E-02	0.952	2.00E-02	0.950	2.16E-02	0.946	2.25E-02	0.944		
36	1.87E-02	0.952	1.95E-02	0.950	2.11E-02	0.946				
37	1.82E-02	0.952	1.90E-02	0.950	2.06E-02	0.946				
38	1.78E-02	0.952	1.86E-02	0.950	2.01E-02	0.946				
39	1.74E-02	0.952	1.81E-02	0.950						
40	1.69E-02	0.952	1.77E-02	0.950						
41	1.65E-02	0.952	1.72E-02	0.950						
42	1.61E-02	0.952	1.68E-02	0.950						
43	1.57E-02	0.952								
44	1.53E-02	0.952								
45	1.50E-02	0.952								
46	1.46E-02	0.952								
$s_{r,n}$	1.43E-02	0.952	1.64E-02	0.950	1.96E-02	0.946	2.19E-02	0.944	2.39E-02	0.942
true min availability	0.952		0.950		0.946		0.944		0.942	
true average cost	69.781		69.210		68.972		69.148		69.481	
efficiency	0.988		0.997		1.000		0.997		0.993	

Table 20. *The minimum average cost and the optimal IRR policy as C_i varies for exp(γ).*

C_i	0.75		1.125		1.5*		1.875		2.25	
availability	0.948		0.948		0.946		0.946		0.946	
average cost	65.632		67.319		68.972*		70.562		72.132	
number of inspection	41		39		38		37		36	
	s_i	θ	s_i	θ	s_i	θ	s_i	θ	s_i	θ
1	5.34E-02	0.948	5.34E-02	0.948	5.55E-02	0.946	5.55E-02	0.946	5.55E-02	0.946
2	4.70E-02	0.948	4.70E-02	0.948	4.89E-02	0.946	4.89E-02	0.946	4.89E-02	0.946
3	4.59E-02	0.948	4.59E-02	0.948	4.77E-02	0.946	4.77E-02	0.946	4.77E-02	0.946
4	4.47E-02	0.948	4.47E-02	0.948	4.65E-02	0.946	4.65E-02	0.946	4.65E-02	0.946
5	4.37E-02	0.948	4.37E-02	0.948	4.54E-02	0.946	4.54E-02	0.946	4.54E-02	0.946
6	4.26E-02	0.948	4.26E-02	0.948	4.43E-02	0.946	4.43E-02	0.946	4.43E-02	0.946
7	4.16E-02	0.948	4.16E-02	0.948	4.32E-02	0.946	4.32E-02	0.946	4.32E-02	0.946
8	4.05E-02	0.948	4.05E-02	0.948	4.21E-02	0.946	4.21E-02	0.946	4.21E-02	0.946
9	3.95E-02	0.948	3.95E-02	0.948	4.11E-02	0.946	4.11E-02	0.946	4.11E-02	0.946
10	3.86E-02	0.948	3.86E-02	0.948	4.01E-02	0.946	4.01E-02	0.946	4.01E-02	0.946
11	3.76E-02	0.948	3.76E-02	0.948	3.91E-02	0.946	3.91E-02	0.946	3.91E-02	0.946
12	3.67E-02	0.948	3.67E-02	0.948	3.82E-02	0.946	3.82E-02	0.946	3.82E-02	0.946
13	3.58E-02	0.948	3.58E-02	0.948	3.73E-02	0.946	3.73E-02	0.946	3.73E-02	0.946
14	3.50E-02	0.948	3.50E-02	0.948	3.63E-02	0.946	3.63E-02	0.946	3.63E-02	0.946
15	3.41E-02	0.948	3.41E-02	0.948	3.55E-02	0.946	3.55E-02	0.946	3.55E-02	0.946
16	3.33E-02	0.948	3.33E-02	0.948	3.46E-02	0.946	3.46E-02	0.946	3.46E-02	0.946
17	3.25E-02	0.948	3.25E-02	0.948	3.37E-02	0.946	3.37E-02	0.946	3.37E-02	0.946
18	3.17E-02	0.948	3.17E-02	0.948	3.29E-02	0.946	3.29E-02	0.946	3.29E-02	0.946
19	3.09E-02	0.948	3.09E-02	0.948	3.21E-02	0.946	3.21E-02	0.946	3.21E-02	0.946
20	3.01E-02	0.948	3.01E-02	0.948	3.13E-02	0.946	3.13E-02	0.946	3.13E-02	0.946
21	2.94E-02	0.948	2.94E-02	0.948	3.06E-02	0.946	3.06E-02	0.946	3.06E-02	0.946
22	2.87E-02	0.948	2.87E-02	0.948	2.98E-02	0.946	2.98E-02	0.946	2.98E-02	0.946
23	2.80E-02	0.948	2.80E-02	0.948	2.91E-02	0.946	2.91E-02	0.946	2.91E-02	0.946
24	2.73E-02	0.948	2.73E-02	0.948	2.84E-02	0.946	2.84E-02	0.946	2.84E-02	0.946
25	2.66E-02	0.948	2.66E-02	0.948	2.77E-02	0.946	2.77E-02	0.946	2.77E-02	0.946
26	2.60E-02	0.948	2.60E-02	0.948	2.70E-02	0.946	2.70E-02	0.946	2.70E-02	0.946
27	2.54E-02	0.948	2.54E-02	0.948	2.64E-02	0.946	2.64E-02	0.946	2.64E-02	0.946
28	2.47E-02	0.948	2.47E-02	0.948	2.57E-02	0.946	2.57E-02	0.946	2.57E-02	0.946
29	2.41E-02	0.948	2.41E-02	0.948	2.51E-02	0.946	2.51E-02	0.946	2.51E-02	0.946
30	2.35E-02	0.948	2.35E-02	0.948	2.45E-02	0.946	2.45E-02	0.946	2.45E-02	0.946
31	2.30E-02	0.948	2.30E-02	0.948	2.39E-02	0.946	2.39E-02	0.946	2.39E-02	0.946
32	2.24E-02	0.948	2.24E-02	0.948	2.33E-02	0.946	2.33E-02	0.946	2.33E-02	0.946
33	2.19E-02	0.948	2.19E-02	0.948	2.27E-02	0.946	2.27E-02	0.946	2.27E-02	0.946
34	2.13E-02	0.948	2.13E-02	0.948	2.22E-02	0.946	2.22E-02	0.946	2.22E-02	0.946
35	2.08E-02	0.948	2.08E-02	0.948	2.16E-02	0.946	2.16E-02	0.946	2.16E-02	0.946
36	2.03E-02	0.948	2.03E-02	0.948	2.11E-02	0.946	2.11E-02	0.946	2.11E-02	0.946
37	1.98E-02	0.948	1.98E-02	0.948	2.06E-02	0.946	2.06E-02	0.946		
38	1.93E-02	0.948	1.93E-02	0.948	2.01E-02	0.946				
39	1.89E-02	0.948	1.89E-02	0.948						
40	1.84E-02	0.948								
41	1.79E-02	0.948								
$s_{r,n}$	1.75E-02	0.948	1.84E-02	0.948	1.96E-02	0.946	2.01E-02	0.946	2.06E-02	0.946
true min availability	0.9480		0.9480		0.9460		0.9460		0.9460	
true average cost	69.032		68.982		68.972		68.980		69.003	
efficiency	0.999		1.000		1.000		1.000		1.000	

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